MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Continued Fractions and Irrationality Measures for Chowla–Selberg Gamma Quotients

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Paper	arXiv:2510.00215v4
Source version	Submitted 12 July, 2026; v1 submitted 30 September, 2025; announced September 2025.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	September 30, 2025
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

No counterexample was found to the continued-fraction identities or to the twenty irrationality-measure bounds. The general continued-fraction asymptotics and denominator estimates are correct. The exact Chowla–Selberg evaluations, and therefore the full lists in Tables 1–2 and Theorems 6.1–6.2, are not able to be verified from the available evidence because a material modular identity is attributed to an unpublished source and all but one of the CM computations are supplied only as table entries.

Theorems 4.1, 4.7, 7.1, 7.3 and Proposition 7.7: General analytic and arithmetic continued-fraction machinery

Pages 7–9 and 19–24 · Sections 4 and 7 · arXiv:2510.00215v4

Correct

The hypergeometric limit formula follows from the recurrence and the asymptotic roots in Proposition 3.1. The explicit polynomial formulas in Theorem 7.1 satisfy the normalized recurrence, Theorem 7.3 controls their denominators prime by prime for $D \in \{2, 3, 4, 6\}$, and Proposition 7.7 gives the required least-common-multiple growth. Combined with Lemma 1.2, this chain yields the stated irrationality-measure formula whenever the exact limit identification in Table 2 has been established.

Verifiable source: Full paper, version 4

Table 2 and Theorem 6.1: The claimed Chowla–Selberg continued-fraction limits are not fully verified

Pages 11–17 · Theorem 5.2, Sections 5.4–6.2, Table 2, and Theorem 6.1 · arXiv:2510.00215v4

Not able to verify

The central claim is that the thirty-nine convergent entries of Table 2 have the displayed Chowla–Selberg limits, including the exact formula for $\mathrm{CS}(-3)$ in Theorem 6.1. Their derivation uses the four functional modular identities in Theorem 5.2 and exact CM values of the associated Hauptmoduln and modular forms. Theorem 5.2 is attributed to reference [2], which the manuscript describes as unpublished, and its proposed differential-equation verification is not carried out. Section 6 works out only tag (1.4) in detail and gives the remaining exact evaluations as table entries. These are concrete nontrivial identities, not consequences recoverable by local substitution alone. No counterexample was found, but neither a complete verification nor an independent proof is supplied here.

Verifiable source: Full paper, version 4

Theorem 6.2: The full list of twenty irrationality measures remains conditional on unverified limit identities

Pages 18 and 24 · Theorem 6.2 and its concluding proof · arXiv:2510.00215v4

Not able to verify

For each entry, the arithmetic comparison of the convergence exponent with the denominator-growth constant correctly gives the displayed upper bound once the continued fraction is known to converge to the stated Chowla–Selberg quotient. That exact target identification is verified in detail only for tag (1.4); the other nineteen bounds depend on table-only CM evaluations and the unavailable input behind Theorem 5.2. Thus the numerical irrationality calculations are internally consistent, but the theorem as a statement about the named gamma quotients is not fully established by the supplied material.

Verifiable source: Full paper, version 4

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The recurrence, asymptotic, denominator, and least-common-multiple arguments are correct and complete. The proof chain identifying the limits with Chowla–Selberg gamma quotients is incomplete for the table as a whole, and its principal functional modular input is not verifiable from the cited unpublished source. One mechanically determined missing factor is reported as a typo.

Theorem 5.2: The functional modular evaluations rely on an unavailable cited proof

Pages 11–12 · Section 5.1 and Theorem 5.2 · arXiv:2510.00215v4

Not able to verify

The four displayed hypergeometric identities are a material input for every exact limit in Table 2. The only proof attribution is to Beukers and Cohen, reference [2], explicitly described as unpublished or in preparation. The text says the identities can be proved by checking a second-order differential equation and initial conditions, but it supplies neither those equations nor the initial-value comparison. The precise unresolved obligation is to prove those four identities on the stated fundamental domains, including the chosen branches. Repair classification: No repair supplied.

Verifiable source: Full paper, version 4

Sections 5.4–6.2 and Tables 1–2: Thirty-eight exact CM-limit computations are omitted

Pages 13–17 · Tables 1–2 and the paragraph introducing Section 6 · arXiv:2510.00215v4

Incomplete as written

The manuscript claims a complete list of rational CM values, converts them into thirty-nine convergent continued fractions, and identifies every limit with an explicit gamma quotient. It then states that only tag (1.4) will be computed in detail and supplies the remaining results only in tables. The general recipe in Section 5.5 does not establish the individual algebraic values of R_N, $D(R_N)$, the modular forms, and their Serre derivatives at the thirty-eight remaining CM points. Those exact identities are indispensable for the named limits and for nineteen entries of Theorem 6.2. Repair classification: No repair supplied; exact derivations or independently checkable symbolic certificates for the omitted entries are required.

Verifiable source: Full paper, version 4

Theorems 7.1 and 7.3; Proposition 7.7: Normalized convergents and denominator growth

Pages 19–24 · Section 7 · arXiv:2510.00215v4

Correct and complete

The explicit formulas satisfy the normalized three-term recurrence and initial data. The prime-adic divisibility estimates cover the four permitted denominators D, and the residue-class density calculation gives the constants m_D^* used at the end of the proof. The resulting application of Lemma 1.2 is valid for every entry once its exact limit and convergence exponent have been established.

Verifiable source: Full paper, version 4

Theorem 4.8: A factor K is missing in the nested continued fraction

Page 9 · Last displayed calculation in the proof of Theorem 4.8 · arXiv:2510.00215v4

Typo

The denominator is printed with the segment $2A - 5^2/(3A - \cdots)$. Replace 5^2 by 5^2K. The preceding definition $U = B - 3^2K/(2B - 5^2K/(3B - \cdots))$ and the coefficient rule $b(n) = -K(2n-1)^2$ uniquely determine the missing factor. The next equality $L = b_0/(a_1 - K/U)$ uses the corrected expression, so the defect does not alter the theorem.

Verifiable source: Full paper, version 4

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 4

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.