MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Bialgebraic varieties of the Gamma function

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Paper	arXiv:2509.24982v2
Source version	Submitted 11 August, 2026; v1 submitted 29 September, 2025; announced September 2025.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	September 29, 2025 (Theorem 1.1); August 11, 2026 (Theorem 1.2)
Paper license	CC BY-NC-SA 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains wrong statements

Both displayed main biconditionals are false as written. Theorem 1.1 permits an arbitrary containing variety W, so its converse fails when W is enlarged. Theorem 1.2 has the same defect, and its requirement $|B| \geq 2$ also excludes the elementary case in which all coordinates vary independently. The intended minimal-closure formulations may be repairable, but they are not the statements printed in version 2.

Theorem 1.1: The converse fails for a nonminimal containing variety W

Page 2 · Theorem 1.1 · arXiv:2509.24982v2

Incorrect

The theorem assumes only $\Gamma(V \cap U_\Gamma^n) \subseteq W$. Take $n = 3$,

$$V = \{(z, z, z) : z \in \mathbb{C}\}, \qquad W = \{(w_1, w_2, w_3) : w_1 = w_2\}.$$

Both varieties are irreducible, V is trivially bialgebraic, and $\Gamma(V \cap U_\Gamma^3) \subseteq W$. Nevertheless, $\dim V = 1$ while $\dim W = 2$. This directly contradicts the claimed implication from trivial bialgebraicity to equality of dimensions. Version 1 avoided this counterexample by defining W to be the Zariski closure of the image. Repair classification: replace the arbitrary W by $\overline{\Gamma(V \cap U_\Gamma^n)}^{\mathrm{Zar}}$, or retain only the proved direction $\dim V = \dim W \Rightarrow V$ trivially bialgebraic.

Verifiable source: Full paper, version 2

Theorem 1.2: The asserted classification fails in both directions as printed

Page 2 · Theorem 1.2 · arXiv:2509.24982v2

Incorrect

First take $n = 1$, $V = \mathbb{C}$, and $W = \mathbb{C}^2$. Then W contains the graph of Γ and $\dim W = 2 = 2 \dim V$, but no partition of $\{1\}$ can satisfy condition (c), which requires $|B| \geq 2$. Thus the forward implication is false; the intended condition must allow $B = \varnothing$. Conversely, take $n = 2$,

$$V = \{(z, z) : z \in \mathbb{C}\}, \qquad W = \{(x_1, x_2, y_1, y_2) : x_1 = x_2\}.$$

With $A = C = \varnothing$ and $B = \{1, 2\}$, the stated structural conditions hold, and W contains the graph over V, but $\dim W = 3 \neq 2 = 2 \dim V$. Hence the reverse implication fails because W may again be enlarged. Repair classification: allow $B = \varnothing$ and require W to be the Zariski closure of the graph over V (or formulate only a one-way result for arbitrary W).

Verifiable source: Full paper, version 2

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proofs do not establish the printed main theorems. Besides the counterexamples to the statements themselves, the final inductions use generic-fiber conclusions at points of the Gamma graph without proving that those points meet the generic locus, and the proof of Theorem 1.2 invokes a higher-dimensional transcendence bound without the needed argument. Earlier curve arguments also apply Proposition 5.1 at a decay scale not supplied by the preceding estimates. Several additional displayed errors are local and are classified as typos only.

Proof of Theorem 1.1: The generic-fiber point used in the induction is not justified

Pages 15–16 · first two paragraphs of the proof of Theorem 1.1 · arXiv:2509.24982v2

Incomplete as written

From $\dim \pi_S(W) = d$, the fiber-dimension theorem gives zero-dimensional fibers above a dense open subset of $\pi_S(W)$. The proof then chooses $c \in U$ and assumes that $\Gamma(c)$ belongs to that open subset. This does not follow for the arbitrary containing variety W in the theorem: the set $\Gamma(U)$ may lie wholly in the exceptional locus. The next sentence similarly upgrades a generic statement to every $c \in U$. If W is instead the Zariski closure of $\Gamma(V)$, density can potentially supply the missing intersection, but that minimality is absent from the printed hypothesis and the necessary argument is not given. The assertion that Γ is locally injective everywhere is also literally false at critical points; only a suitable generic finite-fiber argument is available. Repair classification: no repair is supplied in the paper.

Verifiable source: Full paper, version 2

Proof of Theorem 1.2: The dimension estimate and generic-locus step need additional arguments

Page 16 · proof of Theorem 1.2 · arXiv:2509.24982v2

Incomplete as written

The proof infers

$$2(d-1) \leq \dim \psi_S(W)$$

solely from $\dim \pi_S(V) = d-1$ and the transcendence of the one-variable Gamma function. One-variable transcendence does not by itself yield this higher-dimensional lower bound; an induction hypothesis or a functional-transcendence result must be stated and applied. It then declares the set of c for which $W_{(c,\Gamma(c))}$ has generic dimension to be Zariski dense in $\pi_S(V)$ by the fiber-dimension theorem. That theorem gives a dense open subset in the base $\psi_S(W)$, but it does not ensure that the Gamma graph meets it densely unless an additional minimal-closure argument is made. Finally, the proof presents only the equality-to-structure direction and does not prove the converse for the printed arbitrary W. Repair classification: no complete repair is supplied.

Verifiable source: Full paper, version 2

Claim 6.2 and Proposition 6.4: The available factorial estimate is applied at the wrong input scale

Pages 14–15 · Claim 6.2 and proof of Proposition 6.4 · arXiv:2509.24982v2

Incomplete as written

Proposition 5.1 requires an estimate $|f(n) - m_n| < Ce^{-n}$ in the input index n. Lemma 4.4 and Corollary 4.6 instead give a factorially small error in the index ℓ_n of the nearby negative integer, of order $\mathrm{poly}(\ell_n)/\Gamma(\ell_n)$. For a nonconstant algebraic branch one may have $\ell_n \asymp n^q$ with $0 < q < 1$, so this estimate need not be $O(e^{-n})$. In Claim 6.2 the displayed replacement of $\Gamma(\ell_n)$ by $\Gamma(n)$ is not a consequence of Lemma 4.4; Proposition 6.4 makes the same application without a quantitative comparison at all. A plausible repair is to prove a strengthened version of Proposition 5.1 for errors smaller than every power of n, then combine it with the Puiseux growth $\ell_n \asymp n^q$. That strengthening is not stated or proved here. Repair classification: substantive omitted step; the intended conclusion is not disproved.

Verifiable source: Full paper, version 2

Theorem 1.2(b): The constant-coordinate projection is mislabelled

Page 2 · Theorem 1.2(b) · arXiv:2509.24982v2

Typo

Condition (b) writes $\pi_A : V \to \mathbb{C}^C$ and says that it maps to coordinates indexed by A. The logic of the partition shows that this must be $\pi_C : V \to \mathbb{C}^C$, mapping to coordinates indexed by C. This is a uniquely determined notation correction and does not independently affect the paper's overall correctness status.

Verifiable source: Full paper, version 2

Claim 6.2: The fiber constant is inverted in the invocation of Lemma 4.4

Page 14 · proof of Claim 6.2 · arXiv:2509.24982v2

Typo

At a boundary point $(0, c_j)$ of the inverted curve, the displayed relation is $1/\Gamma(\alpha(-n)) = c_j$, hence $\Gamma(\alpha(-n)) = 1/c_j$. Lemma 4.4 must therefore be invoked with $1/c_j$, not c_j. Its local constant becomes proportional to $|c_j|$, so one should use $\max_{c_j \neq 0} |c_j|$ rather than $1/\min_j |c_j|$; the printed minimum may also vanish when some $c_j = 0$. These constants can be corrected mechanically, although the separate input-scale gap reported above remains.

Verifiable source: Full paper, version 2

Proposition 5.1: A basepoint is dropped from the last finite-difference display

Page 13 · proof of Proposition 5.1 · arXiv:2509.24982v2

Typo

After correctly writing the terms as $(f - q)(r + \ell)$, the next display changes them to $(f - q)(\ell)$. The same recurrence works with $(f - q)(r + \ell)$ throughout. This is a local index mismatch with a uniquely determined correction and does not affect the overall status by itself.

Verifiable source: Full paper, version 2

Corollary 3.3: The leading constant in the limit computation is missing an exponent

Page 6 · proof of Corollary 3.3 · arXiv:2509.24982v2

Typo

For nonzero integer a, the leading term of the multiplier $m_a(az+b)$ is $a^a z^a$. The displayed limit therefore yields $a^{ar_2}=1$, rather than $a^{r_2}=1$. Because $a \in \mathbb{Z} \setminus \{0\}$ and $r_2 \neq 0$, either relation gives the required conclusion $|a|=1$. This is a harmless omitted exponent and does not alter the argument's conclusion.

Verifiable source: Full paper, version 2

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 2

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.