MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Data-Driven State Observers for Measure-Preserving Systems

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Paper	arXiv:2509.20596v2
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Prompt	Standardized prompt
Novelty cutoff	September 24, 2025
Paper license	CC BY-NC-SA 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains wrong statements

The claimed injectivity of every deep KKL map under uniform backward distinguishability is false, as is the resulting observer representation with a left inverse. An explicit smooth measure-preserving interval system satisfies all stated assumptions but makes the one-component KKL map noninjective. The three learning guarantees are not verified because their proofs do not establish the printed uniform conclusions.

Theorem 1: Uniform backward distinguishability does not force the deep KKL map to be injective

Page 10 · Theorem 1 · arXiv:2509.20596v2

Incorrect

Take $\mathbb{X} = [-1, 1]$, $f(x) = -x$, $h(x) = e^x$, observer order $m = 1$, and $\beta = 1/2$. The map f is invertible with continuous inverse, h is bounded, and Assumption 4 holds because $h(f^{-1}(x)) = e^{-x}$ is injective. Yet the series in Definition 5 gives

$$\zeta_0(\beta, x) = \frac{e^{-x} + \beta e^x}{1 + \beta}.$$

If $x_0 = (\log 2)/2$ and $x_\pm = x_0 \pm 1/4$, then $x_- \neq x_+$ while $\zeta_0(1/2, x_-) = \zeta_0(1/2, x_+)$. Thus every hypothesis of Theorem 1 holds and its conclusion fails. Repair classification: No repair supplied; stronger assumptions controlling cancellation among the delayed observations are necessary.

Theorem 2: The asserted observer with a left pseudo-inverse need not exist

Pages 10–11 · Theorem 2 and equations (13)–(14) · arXiv:2509.20596v2

Incorrect

The same counterexample satisfies Assumption 1, with $|\det Df| = 1$; Assumption 2, using normalized Lebesgue measure; Assumption 3 for every finite Sobolev order; and Assumption 4. For $\beta = 1/2$, however, ζ is not injective, so no left pseudo-inverse $\zeta^\dagger$ exists on $\zeta(\mathbb{X})$. Consequently the displayed state reconstruction in (13) is false under the stated assumptions. The Koopman-series identity by itself does not restore injectivity.

Theorems 3 and 4: The uniform KRR error bounds are not established

Pages 14–17 · Theorems 3 and 4 · arXiv:2509.20596v2

Not able to verify

These theorems add hypotheses under which a Lipschitz extension of $\zeta^\dagger$ is assumed to exist, so the counterexample to Theorem 1 alone does not disprove their restricted implications. Their proofs nevertheless leave two nontrivial obligations unresolved: the cited KRR theorem applies to the marginal distribution of the actual noisy inputs $\widetilde{\zeta}(x)$, whereas condition (iv) is imposed on the integral operator for $\nu_z = \zeta_\sharp \nu$; and the derived sup-norm estimate is stated only on $\widetilde{\zeta}(\mathbb{X})$, while (20) evaluates $\widehat{\zeta}^\dagger$ at $\zeta(x)$. Lipschitz continuity is assumed for $\zeta^\dagger$, not for the learned map, so proximity of these two input sets does not close the latter step. No independent proof of (20) was supplied.

Theorem 5: The snapshot guarantee is not established

Pages 18–20 · Lemmas 3–4 and Theorem 5 · arXiv:2509.20596v2

Not able to verify

The spectral construction used to justify Algorithm 3 contains false intermediate claims. In Lemma 3, the printed radius is an infimum over a set containing λ and is therefore zero; moreover, an L^2 spectral projection need not preserve H^s. Lemma 4 assumes that every fine arc supplies a small-residual vector and calls a Gram matrix unitary, neither of which follows. For the identity Koopman operator, arcs away from 1 have no spectral mass and every normalized vector has residual $|1 - \lambda|$, directly contradicting the asserted mesh-scale construction. The theorem also inherits the unresolved KRR transfer in Theorem 4, so its conclusion remains unverified rather than proved false.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The injectivity proof is contradicted by an explicit example and reverses the conclusion of its own empty-set assertion. The learning proofs misapply the cited KRR estimate to a different input distribution and do not transfer a bound from the approximate-input set to the exact-input set. The snapshot proof additionally relies on false spectral lemmas.

Proof of Theorem 1: Finite derivative equalities at β are incorrectly transported to $\beta = 0$

Page 10 · proof of Theorem 1 · arXiv:2509.20596v2

Incorrect as written

Backward distinguishability identifies derivatives of the holomorphic difference at $\beta = 0$. The proof instead assumes that the first m derivatives vanish at an arbitrary fixed β and then says that evaluation at $\beta = 0$ gives the same information. A finite jet at one point does not determine a holomorphic function's finite jet at another point. The sets $\mathbb{K}_\ell$ therefore are not shown to be empty; the final paragraph also literally changes $\mathbb{K}_m = \varnothing$ into the existence of a distinct pair. The explicit interval counterexample in Part 1 shows that this is not a repairable omission under the printed hypotheses.

Proofs of Theorems 3 and 4: The cited learning estimate is applied to the wrong input law and on the wrong evaluation set

Pages 14–17 · proofs of Theorems 3 and 4 · arXiv:2509.20596v2

Incomplete as written

Smale–Zhou's estimate is formulated for independent pairs drawn from one fixed joint law and its integral operator is defined using that law's input marginal. Here the KRR inputs are $\widetilde{\zeta}(x)$, but condition (iv) uses the different marginal $\zeta_\sharp\nu$. No comparison of the two integral operators or their ranges is proved. Even granting an estimate on $\widetilde{\zeta}(\mathbb{X})$, the last triangle argument controls $\widehat{\zeta}^\dagger(z)$ for a nearby $z \in \widetilde{\zeta}(\mathbb{X})$, not the printed quantity $\widehat{\zeta}^\dagger(\zeta(x))$; no modulus of continuity for the learned function is provided. Repair classification: No repair supplied.

Verifiable source: Smale–Zhou, learning-theory sampling framework

Lemmas 3 and 4: The approximate-eigenfunction spanning argument is false

Pages 18–20 · Lemmas 3 and 4 · arXiv:2509.20596v2

Incorrect as written

Because $\lambda \in \mathbb{J}$, the radius $\inf_{\lambda' \in \mathbb{J}} |\lambda' - \lambda|$ in Lemma 3 equals zero, which would assert an H^s eigenfunction at every spectral point; continuous spectrum need not have eigenfunctions. Replacing infimum by supremum does not repair the proof, since $E'(\mathbb{J})g$ need not lie in H^s. Lemma 4 then assigns mesh-scale residuals to every equally spaced grid point even if its arc has zero spectral projection. The identity operator gives a direct obstruction: away from $\lambda = 1$, the residual is exactly $|1 - \lambda|$ for every unit vector. The subsequent assertion that the Gram matrix is unitary is also false. Repair classification: No repair supplied; a new finite-dimensional spectral approximation theorem with explicit support and regularity hypotheses is required.

Equation (19): The displayed KRR coefficient formula contains an extra vector factor

Page 14 · equation (19) · arXiv:2509.20596v2

Typo

The printed matrix $G_\kappa c_k + \alpha I$ is dimensionally undefined. Differentiating the objective displayed immediately above gives $\widehat{c}_k = (G_\kappa + (\alpha/2)I)^{-1} x_k$. The intended correction is mechanically determined and does not affect the substantive audit findings.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2509.20596v2

2. Smale–Zhou, learning-theory sampling framework

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.