MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

On Conservative Matrix Fields: Continuous Asymptotics and Arithmetic

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Paper	arXiv:2507.08138v2
Source version	Submitted 15 July, 2026; v1 submitted 10 July, 2025; announced July 2025.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	July 10, 2025; July 15, 2026 for results first added in version 2
Paper license	CC BY-SA 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	**Statements**	**Contains wrong statements**
02	**Proofs**	**Contains incorrect or incomplete proofs**
03	**Novelty**	**No non-novelty findings**

01 / STATEMENTS

Statements

Contains wrong statements

The D-finite construction of conservative matrix fields, the D-finiteness of their coefficients, and the local limiting-matrix results are correct. Two central asymptotic assertions are false under their printed hypotheses. Lemma 4.3 and Corollary 4.4 omit control of singular evaluated factors in the finite prefix, and Theorem 5.1 omits the strict spectral-modulus gap needed for convergence. Explicit rank-one and rank-two counterexamples isolate both defects. The two conjectures in Section 6 are presented as conjectures and are not treated as proved claims.

Theorem 3.1 and Lemma 3.4: D-finite functions generate CMFs with D-finite coefficients

Pages 11 and 13 · Theorem 3.1 and Lemma 3.4 · arXiv:2507.08138v2

Correct

For a basis B of the finite-dimensional Ore-module image, the identity $BM_v^f = \sigma_v(B)$ determines an invertible basis-change matrix. Applying two shifts gives $BM_v^f \sigma_v(M_w^f) = \sigma_{v+w}(B)$, which is exactly the cocycle relation. For a matrix coefficient $e_i^\top M_v(x) e_j$, every shift is a rational-function linear combination of the r entries of $e_i^\top M_v(x)$; hence its shift-module dimension is at most r. Linear combinations cover arbitrary rational vectors.

Verifiable source: Full paper, version 2

Proposition 4.1, Corollary 4.2, and Proposition 4.5: Limiting generators, trajectory expansion, and normalized spectrum

Pages 15 and 17–18 · Proposition 4.1, Corollary 4.2, and Proposition 4.5 · arXiv:2507.08138v2

Correct

Away from zeros of the leading denominator forms, every balanced rational entry has a constant term, a $1/n$ term, and an $O(n^{-2})$ remainder. The bounded inverse generators force the limiting matrices to be invertible, and the cocycle equation makes them commute. Multiplying the finitely many generator expansions gives Corollary 4.2. When the commuting limiting generators are diagonalizable, they are simultaneously diagonalizable over $\mathbb{C}$; the logarithms of the absolute values of the joint eigenvalues give the asserted continuous normalized spectrum, understood with a continuous local labeling or as an unordered spectrum.

Verifiable source: Full paper, version 2

Lemma 4.3 and Corollary 4.4: A singular finite factor defeats the claimed invertible factorization

Pages 16–17 · Lemma 4.3, Equations (14)–(16), and Corollary 4.4 · arXiv:2507.08138v2

Incorrect

In rank one, take $T(n) = (n-1)/n$. It is a nonzero element of $\mathrm{GL}_1(\mathbb{Q}(n))$, satisfies $T(n) = 1 - 1/n$, and its limiting eigenvalue is 1, but $P(n) = T(1) \cdots T(n) = 0$ for every $n \geq 1$. It therefore cannot equal $B\, 1^n n^{-1}(1 + o(1))A$ with $A, B \in \mathrm{GL}_1(\mathbb{R})$. The same defect occurs inside the CMF class. Define the balanced rank-one CMF $M_v(x) = x/(x+v)$. The cocycle identity holds exactly, but for $x = 0$ and direction $v = 1$ one has $M_n(0) = 0$ for every $n \geq 1$, while the limiting trajectory eigenvalue is 1. Thus Corollary 4.4 is also false as stated. Repair: require the evaluated finite prefix to be defined and invertible, start the Levinson reduction after a sufficiently large index, and absorb that nonsingular prefix into B.

Verifiable source: Full paper, version 2

Theorem 5.1: Proposition 4.5 does not supply the spectral gap required for convergence

Pages 19–20 · Theorem 5.1 and the sentence following its proof · arXiv:2507.08138v2

Incorrect

The theorem assumes only Proposition 4.5, which permits eigenvalues of equal modulus, but its proof invokes Equation (17), derived under the strict inequalities of Corollary 4.4. A concrete counterexample is the constant balanced rank-two CMF generated by $T = S\,\mathrm{diag}(1,-1)S^{-1}$ with $S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$. It has the exact factorization $T^n = S\,\mathrm{diag}(1,(-1)^n)S^{-1}$. With right vector e_2, $p = (1,0)^\mathsf{T}$, and $q = (1,2)^\mathsf{T}$, all three displayed conditions hold with $k = 1$, $j = 2$, and $c = 1/3$, and both coordinates of $S^{-1}e_2$ are nonzero. Nevertheless, the CMF ratio equals 0 for even n and 1 for odd n, so it does not converge to c. Repair: assume the strict modulus ordering from Corollary 4.4, together with its corrected evaluated-invertibility condition.

Verifiable source: Full paper, version 2

Proposition 6.1: Homogeneity and rank-two continuity under the stated existence assumptions

Pages 20–21 · Proposition 6.1 · arXiv:2507.08138v2

Correct

The identity $L^{p,q}_{x,kv}(n) = L^{p,q}_{x,v}(kn)$ gives $l(kv) = l(v)$, $\rho(kv) = k\rho(v)$, $\eta(kv) = k\eta(v)$, and $\delta(kv) = \delta(v)$ directly from the definitions when the listed limits exist. In rank two, linear independence forces the leading term of the numerator-denominator cancellation to use the other eigenvalue, so the normalized convergence rate is the difference of the two normalized logarithmic moduli and is continuous on a region with the required spectral separation.

Verifiable source: Full paper, version 2

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proof of Lemma 4.3 incorrectly starts all products and transformations at index 1 and asserts a nonzero scalar-product constant that may vanish. This creates a genuine counterexample and propagates to Corollary 4.4. Theorem 5.1 then applies Equation (17) without assuming its strict spectral-gap hypotheses. Both proof chains have verified repairs after adding the missing restrictions. A coefficient-ring notation error is reported separately in yellow.

Lemma 4.3: The proof cannot absorb a singular initial factor

Pages 16–17 · Equations (14)–(16) · arXiv:2507.08138v2

Incorrect as written · verified repair

The argument uses $H(1)^{-1}$ and the diagonal product $\prod_{k=1}^{n}(\lambda_i + d_i/k)$ without proving that these factors are invertible. The assertion that its constant c_i is nonzero fails whenever d_i/λ_i is a negative integer; $T(n) = (n-1)/n$ realizes the failure at $k = 1$. Field invertibility in $\mathbb{Q}(n)$ does not imply invertibility after evaluation at every positive integer, a distinction already noted in Definition 11. The repair is complete after adding that a finite evaluated prefix is defined and invertible: choose N so large that $H(n)$ and every $\lambda_i + d_i/n$ are invertible for $n \geq N$, apply the summable-perturbation theorem to the tail product, use $\prod_{k=N}^{n}(1 + d_i/(\lambda_i k)) = c_{i,N} n^{d_i/\lambda_i}(1 + o(1))$ with $c_{i,N} \neq 0$, and absorb the nonsingular prefix into B.

Verifiable source: Benzaid–Lutz asymptotic framework

Corollary 4.4: Symbolic CMF invertibility is insufficient after specialization

Page 17 · Application of Lemma 4.3 immediately before Equation (17) · arXiv:2507.08138v2

Incorrect as written · verified repair

A trajectory matrix belongs to $\mathrm{GL}_r(\mathbb{Q}(n))$ symbolically, yet an evaluated early factor may be singular or undefined. Then the finite product cannot be absorbed into an invertible $B(x, v)$. The rank-one CMF $M_v(x) = x/(x + v)$ at $x = 0$, $v = 1$ gives an explicit singular prefix and identically zero products. Add the hypothesis that the relevant evaluated trajectory factors, or equivalently one complete prefix through a sufficiently large nonsingular tail, are defined and invertible. The corrected Lemma 4.3 then proves the stated factorization.

Verifiable source: Full paper, version 2

Theorem 5.1: Equation (17) is invoked outside its stated spectral regime

Pages 19–20 · Opening hypothesis and final use of Equation (17) · arXiv:2507.08138v2

Incorrect as written · verified repair

The opening reference to Proposition 4.5 supplies diagonalizable commuting limiting generators but not $|\lambda_1| > \cdots > |\lambda_r|$. Equation (17), on which every Θ estimate in the proof rests, was obtained only from Corollary 4.4 under exactly that strict ordering. The equal-modulus matrix counterexample in the statement finding shows that this is not an expositional omission. Replace the opening hypothesis by the corrected hypotheses of Corollary 4.4 together with those of Proposition 4.5. Then $|\lambda_j/\lambda_k| < 1$, Equation (17) applies to the denominator and the cancelled numerator, and the displayed convergence rate follows.

Verifiable source: Full paper, version 2

Remark 3: The annihilating-operator coefficient field is abbreviated incorrectly

Page 4 · First two bullets of Remark 3 · arXiv:2507.08138v2

Typo

The equivalence is printed using $K[\partial_i] \cap \operatorname{ann}(f)$, and the next bullet likewise places the minimal shift operator in $K[S_{x_i}]$. Replace these by $K(x, z)[\partial_i] \cap \operatorname{ann}(f)$ and $K(x, z)[S_{x_i}]$, respectively. The displayed coefficients $p_j \in K(x, z)$ and every subsequent calculation already use the rational-function coefficient field, so the intended correction is unique and no main argument changes.

Verifiable source: Full paper, version 2

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2507.08138v2
2. Full paper, version 2
3. Benzaid–Lutz asymptotic framework

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ *is proved for every sufficiently small delta* > 0 *and* P *is immediately monotone in the required direction, then the claim that* $P(\text{delta})$ *holds for every delta* > 0 *requires no correction; the omitted monotonicity step is exposition rather than an audit finding.*

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ *and subsequently applied at* $t = 0$ *remains a formal scope mismatch unless the endpoint case was separately established before the application.*

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.