

Correctness, proof completeness, and novelty audit

Distribution of Farey fractions with k -free denominators

Bittu Chahal, Tapas Chatterjee, Sneha Chaubey

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Paper license	See the arXiv abstract page for this version
Scope	Statements, proofs, and novelty only

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SUMMARY

01	Statements	Suspected incorrect statements
02	Proofs	Suspected incomplete or incorrect proofs
03	Novelty	No novelty concerns found

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01 / STATEMENTS

Statements

Suspected incorrect statements

The uniform-frequency bound, the second-moment identity outside the residue class one, and the displayed pair-correlation formula are false. Equidistribution, discrepancy order, the joint-family GRH criterion, moment bounds and correlation existence survive the corrections described below.

Theorem 1.1: The asserted uniform Weyl bound fails at a common denominator

PDF p. 3 · Theorem 1.1; proof (25), p. 24 · arXiv:2507.00228v3

Suspected incorrect statement

The theorem states the bound with a constant depending on m , not on the nonzero frequency r . Fix $m = b = 1$ and any $k \geq 2$, and take $r = Q!$. Every denominator in the Farey set divides r , so every summand $e(r\gamma)$ equals one. The sum is therefore $N(Q, k, 1) \sim \mathcal{C}(k, 1)Q^2$, with $\mathcal{C}(k, 1) > 0$. The claimed right side is $O(Q^2 \exp(-c(\log Q)^{3/5}/(\log \log Q)^{1/5})) = o(Q^2)$, a contradiction. The fixed-frequency bound does not have this counterexample, but is weaker than the stated uniform minimum. The proof loses the dependence of the cancellation factor on Q/q before (25). A verified safe uniform form replaces the minimum by $\min\{Q^2, |r|^\varepsilon Q \exp(-c'(\log Q)^{3/5}/(\log \log Q)^{1/5})\}$, with a suitably smaller positive c' . For the second bound split frequencies according as $\log |r|$ exceeds a suitable multiple of the decay exponent: in the large case a divisor bound absorbs that decay into $|r|^\varepsilon$, and in the small case every divisor $q \mid r$ is below $Q^{1/2}$, where the weighted Mobius estimate retains uniform decay. This changes the stated uniform bound; it is not a proof of the original formula.

Theorem 1.4 exact identity: The constant term is wrong when the Farey set excludes one

PDF p. 4 · first displayed identity in Theorem 1.4; Lemma 2.6, p. 15; (30), p. 26 · arXiv:2507.00228v3

Suspected incorrect statement

Take $m = 3$, $b = 2$, $Q = 2$ and any $k \geq 2$. The set is $\{1/2\}$, its cardinality is one, and the left side is $1/4$. On the right, $M_1(2) = -1$ and $M_2(1) = 1$, so the double sum is $1 + 1 - 2(1/2) = 1$. The printed right side is zero. This is not only a small-order exception. Put $\eta = 1$ when $b \equiv 1 \pmod{m}$ and zero otherwise. Mobius inversion gives $\sum_q M_q(Q/q) = \eta$, and the sawtooth sum between successive fractions is $Nu - v - \eta/2$. If $\eta = 0$, reflection symmetry gives $\sum_j (\gamma_j - j/N) = -1/2$ and direct integration gives $I = N \sum_j (\gamma_j - j/N)^2 - 1/6$. If $\eta = 1$, the printed relation $I = N \sum_j (\gamma_j - j/N)^2 + 1/12$ is valid. Thus a verified repair replaces the final -1 in the theorem's numerator by $2 - 3\eta$. For every non-one residue class the error in the stated formula is exactly $1/(4N)$. The asymptotic bounds are not invalidated by this correction.

Theorem 1.7: The displayed pair density omits the modulus factor

PDF p. 8 · (9) and the explicit formula for $F_k(n)$; Lemma 2.8, p. 16; (59)–(60), pp. 42–43 · arXiv:2507.00228v3

Suspected incorrect statement

The local factor in Lemma 2.8 for $r_1 = r_2 = m$ includes

$$D_m = \frac{\varphi(m)^2}{m^2} \prod_{p|m} (1 - p^{-2})^{-1}.$$

Formula (60) still defines F_k using that full factor, but the explicit F_k in the theorem omits it while retaining $\varphi(m)^{-2}$ outside. A decisive regime is $m = 2$ and $\mathcal{C}(k, 2) < \lambda < 2\mathcal{C}(k, 2)$, where only $n = \delta = d_1 = d_2 = r = 1$ contributes. Write $A_k = \prod_{p \neq 2} (1 - 2/(p^{k-1}(p+1))) > 0$. The density of coprime, odd, k -free denominator pairs is

$$\frac{1}{4} \prod_{p \neq 2} (1 - p^{-2}) \left(1 - \frac{2}{p^{k-1}(p+1)} \right) = \frac{2}{\pi^2} A_k.$$

The theorem instead uses $6A_k/\pi^2$, making its first nonzero density three times the actual density. The same change of variables as in (59) yields the positive multiplier $\lambda^{-2} \log(\lambda/\mathcal{C}(k, 2))$, so the discrepancy survives in the limiting correlation measure, not only in an intermediate count. Independent exact-fraction enumerations for $k = m = 2$ on raw differences $(1.2/Q^2, 1.8/Q^2)$ give normalized counts 0.088091, 0.087518 and 0.085402 at $Q = 300, 600, 1200$; the correctly normalized limit is approximately 0.086346, whereas the printed formula gives approximately 0.259038. The local-density argument, rather than these finite samples, proves the contradiction.

Theorem 1.7, including the unrestricted modulus case: Non- k -free coefficient tuples are also included incorrectly

PDF p. 42 · application of Lemma 2.8 with $\delta_1 = d_1\delta$, $\delta_2 = d_2\delta$; (60) · arXiv:2507.00228v3

Suspected incorrect statement

Lemma 2.8 assumes both δ_1 and δ_2 are k -free. The proof applies it to $d_1\delta, d_2\delta$ without those restrictions. For $k = 2$, $\delta = d_1 = d_2 = 2$ and $r = 1$, the original summand in (57) is identically zero since $4 \mid q_i d_i \delta$. Nevertheless the extended Euler product in the theorem has local factor at two equal to $1 - (4+4)/(2(2+1)) = -1/3$. This affects $F_2(8)$ even for $m = 1$. For indices below eight there is no such nonzero erroneous tuple; all other non-square-free tuples at eight have a zero Euler factor already. Hence on $8\mathcal{C}(2, 1) < \lambda < 9\mathcal{C}(2, 1)$ the missing contribution is exactly a positive multiple of $\lambda^{-2} \log(\lambda/(8\mathcal{C}(2, 1)))$, proving a discrepancy independently of the modulus-factor defect. A verified repair of the coefficient sum retains only $d_1\delta, d_2\delta$ that are k -free and retains D_m . The support condition bounds all four coefficient variables independently of Q ; Lemma 2.8 is then applicable with constants allowed to depend on those finitely many fixed twists. This establishes existence and a corrected density, not the printed formula.

Corollary 1.2, Theorems 1.3 and 1.5, and moment bounds: The global distribution conclusions have verified arguments

PDF pp. 3–5 and 24–30 · arXiv:2507.00228v3

No issue found

For each fixed nonzero integer r , the exact Ramanujan-sum expansion gives $|\sum_{\gamma} e(r\gamma)| \leq Q\tau(|r|)$. With $N \asymp_m Q^2$, this already proves equidistribution without the false uniform minimum. The discrepancy argument uses $|M_q(Q/q)| \ll_m (Q/q)W(q)E(Q/q)$, where $W(q) = \prod_{p|q} (1 - p^{-1/2})^{-1}$ and a weakened cancellation factor $E(x) = \exp(-c\sqrt{\log x})$ suffices. The mean bound $\sum_{q \leq x} W(q)\mu_k(q)^2 \ll x$ and a dyadic split give $\sum_q |M_q(Q/q)| \ll_m Q$, hence discrepancy $O_m(1/Q)$; the first or last fraction-free interval supplies the lower order. Under GRH the weighted Mobius bound gives $I \ll Q^{1+\epsilon}$, and the endpoint correction above is bounded, so Cauchy–Schwarz gives the claimed first-moment estimate. Read as a criterion for the family over reduced residue classes, the converse supplies every $M_b(x)$ bound; character orthogonality then gives $\sum_{n \leq x} \mu(n)\chi(n) = O(x^{1/2+\epsilon})$, the reciprocal- L holomorphy criterion for GRH. Taking all moduli gives the full Dirichlet GRH criterion. The unconditional second-moment bound also follows directly from the

weighted Mobius estimate: group the double sum by $\gcd(q_1, q_2) = \delta$, use $W(\delta a) \leq W(\delta)W(a)$ and split a and δ at square roots. The tails are power-saving and the remaining terms retain the stated subexponential decay. Thus the erroneous Proposition 2.3 below is unnecessary for that bound.

Theorem 1.6 and correlation existence: The lattice method supports the existence assertions

PDF pp. 6–7 and 20–23, 31–39 · arXiv:2507.00228v3

No issue found

For $\nu < k$, the Fourier expansion and Poisson summation reduce the statistic to finitely many coefficient tuples for each compact test-function support. Mobius inversion detects both coprimality and all k -free linear forms. The lattice has rank two, and because its index involves only primes dividing the d_j , it is coprime to m in the contributing terms; consequently the reduction modulo m is invertible and the congruence density is the stated $\#A_{m,r}/m^2$. The determinant lower bound gives an absolutely convergent majorant with Euler factors $1 + p^{-2} + (2^\nu - 1)p^{-k} + (2^\nu - 1)p^{-k-1}$. For the convex polygonal regions used here, elementary weighted lattice counting, with an integral lattice basis chosen in reduced form, gives the summed error $O(Q^{1+\nu/k} \log Q)$; divided by $N \asymp Q^2$ it tends to zero exactly under $\nu < k$. The final summation bounds require the index correction recorded below. The undefined extra Ψ_k condition in the intermediate enlarged region on p. 38 must be omitted: all arithmetic k -free restrictions are already imposed by the Mobius weights, and the ensuing region and final formula contain no such real-variable condition. With that local notation correction, the support argument justifies the region enlargement. The pair case for $k = 2$ uses its separate proof after the two corrections above. Finally, the minimum nonzero normalized spacing tends to the positive constant $\mathcal{C}(k, m)$, so none of these correlation measures is Poissonian.

02 / PROOFS

Proofs

Suspected incomplete or incorrect proofs

The false formulas arise from a lost frequency dependence, a missing endpoint term and two incorrect local-density manipulations. A separate harmonic-sum proposition also misses a principal-character normalization and nonprincipal residues.

Theorem 1.1 proof: The decay cannot be made uniform over all divisor sizes

PDF p. 24 · line preceding and display (25) · arXiv:2507.00228v3

Suspected incorrect as written

Proposition 2.2 is applied at $x = Q/q$. Its decay is controlled by $\log(Q/q)$, which can remain bounded when q is comparable to Q . Replacing it by decay in $\log Q$ loses the required frequency dependence; summing over all divisors of a common multiple gives the counterexample in Part 1. Verified repairs for fixed frequency and for a differently capped uniform estimate are given there. Neither proves the printed uniform bound.

Lemma 2.6 and the moment identity: The sawtooth constant depends on whether one is included

PDF pp. 15 and 25–27 · Lemma 2.6 and (30) · arXiv:2507.00228v3

Suspected incorrect as written

The final constant in the proof is $-\frac{1}{2} \sum_q M_q(Q/q)$, not always $-1/2$. By Lemma 2.5 with the constant function at the endpoint, the sum of these coefficients is $\eta = \mathbf{1}_{b \equiv 1 \pmod{m}}$. The verified sawtooth and integrated corrections are supplied in Part 1. They repair every first- and second-moment estimate because the change is $O(1)$ in I and $O(1/N)$ in the moment; they do not leave the exact central identity unchanged.

Proposition 2.3: The harmonic progression main term has the wrong normalization and omits constants

PDF pp. 13–14 · formula for $\mathcal{M}_{m,d,l}$ and nonprincipal-character contour step · arXiv:2507.00228v3

Suspected incorrect as written

Already for $d = l = 1$, the correct coefficient of $\log x$ in $\sum_{n \leq x, n \equiv b \pmod{m}} \mu_k(n)^2/n$ is $1/(mL(k, \chi_0))$. Character orthogonality contributes $1/\varphi(m)$ and the principal $L(s+1, \chi_0)$ residue is $\varphi(m)/m$. The displayed formula instead has coefficient $\varphi(m)/(mL(k, \chi_0))$; for $m = 3$ it is twice the correct leading term. Moreover, for each nonprincipal character the Perron integrand $F(s)x^s/s$ generally has residue $F(0)$ at zero even though $F(s)$ itself is regular. These constants depend on b and are omitted. A correct asymptotic must divide the principal main term by $\varphi(m)$ and add the nonprincipal $\bar{\chi}(b)F_\chi(0)/\varphi(m)$ contributions. The second-moment upper bounds have the independent verified repair in Part 1, so no falsehood of those bounds is inferred from this proposition.

Theorem 1.7 proof: The two local-density errors propagate into the central formula

PDF pp. 41–43 · (57)–(62), especially (59)–(60) · arXiv:2507.00228v3

Suspected incorrect as written

The modulus factor D_m in the valid $P_{m,m}^k$ calculation is absent from the explicit theorem, and the application of Lemma 2.8 includes twists outside its k -free hypotheses. The counterexamples and exact affected regimes are given in Part 1. Verified repair: restrict the coefficient sum to $\mu_k(d_1\delta)^2 = \mu_k(d_2\delta)^2 = 1$ and use the complete $P_{m,m}^k$ factor. For every compact support, $\delta d_1 d_2 r < \Lambda/\mathcal{C}(k, m)$ bounds these twists independently of Q , so their individual lattice errors and nonprincipal character-sum errors are $o(Q^2)$ after summation. The integral change of variables contributes the printed logarithmic kernel and no further normalization. This gives a valid limiting density with corrected coefficients, while (9) as printed remains false.

Theorem 1.6 summation bounds: The bounds assigned to the two coefficient indices are interchanged

PDF p. 7 · Theorem 1.6; pp. 34–35, 39 · arXiv:2507.00228v3

Suspected typo

The theorem prints $A_j \leq (\nu - 1)C(\Lambda, k, m)^2$ and $B_j \leq \nu C(\Lambda, k, m)^2$. The proof defines $A_j = n_j e_j$ and $B_j = n_j \Delta_j$, with $e_j \leq \nu C(\Lambda, k, m)$, $\Delta_j \leq (\nu - 1)C(\Lambda, k, m)$ and $n_j \leq C(\Lambda, k, m)$. Thus the mechanically determined bounds are $A_j \leq \nu C(\Lambda, k, m)^2$ and $B_j \leq (\nu - 1)C(\Lambda, k, m)^2$. Interchanging these two index bounds makes the final expression match the established coefficient ranges and leaves the existence proof unchanged. In the intermediate region on p. 38, the extra undefined condition $\Psi_k(n_j(ye_j - x\Delta_j)) = 1$ is also extraneous; removing it gives precisely the enlarged region used in the next lines, with arithmetic restrictions already included in the coefficient weights.

Theorem 1.5 range: The discrepancy requires a nonempty set

PDF p. 5 · phrase for all orders in Theorem 1.5 · arXiv:2507.00228v3

Suspected minor formal issue

When $m = 3$, $b = 2$ and $Q = 1$, the Farey set is empty, so the displayed discrepancy with normalization by its cardinality is undefined. State the assertion for orders with $N(Q, k, m) > 0$, or for all sufficiently large Q with fixed m, b . This local range correction is consistent with every application, all of which concern the limit as $Q \rightarrow \infty$; the verified upper and lower discrepancy orders are unchanged.

Remaining global and lattice inputs: The valid ingredients and limiting ranges are checked separately

PDF pp. 9–12, 14–24 and 31–39 · arXiv:2507.00228v3

No issue found in the reviewed proof

The cardinality Euler product has its principal pole at two and gives the constant (8). The weighted Mobius series is a reciprocal Dirichlet L -function times finite Euler factors, so the fixed-modulus zero-free region in [Khale, Theorem 1.1](#) supplies the unconditional cancellation; the fixed-modulus small-height range is absorbed by choosing a smaller constant. Under GRH, holomorphy and standard reciprocal- L growth in $\Re s > 1/2$ supply the stated weakened bound. Here analytic continuation, not the incorrect phrase absolute convergence in that half-plane, is what the contour argument uses. The exact Farey expansion in Lemma 2.5 and its Ramanujan-sum specialization are valid. The sawtooth integral is $\gcd(q_1, q_2)^2/(12q_1q_2)$; grouping by the gcd gives convergent sums under the GRH exponent. In the correlation proof, the Poisson divisibility equations and support bounds agree with the underlying method of [Boca-Zaharescu, §§3–4](#). Absolute convergence of the lattice coefficient sum follows from the explicit local majorant above. The limiting errors vanish for $\nu < k$; the separate pair argument has error $O(Q^{-1+1/k+\varepsilon} \log^2 Q)$ after normalization, which tends to zero for every $k \geq 2$.

03 / NOVELTY

Novelty

No novelty concerns found

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Suspected typographical errors and minor formal issues remain fully visible in yellow with their locations and proposed repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs result from green (no issues found).

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: [arXiv:2507.00228v3](#)

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Do not investigate, infer, or comment on authorship, contribution credit, acknowledgments, changes in author lists between versions or sources, collaboration history, or research conduct. These matters are outside the audit and must not appear in any finding, summary, source assessment, or report text. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
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- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only comparison of central mathematical results with earlier papers. First identify only the central results that the reviewed paper genuinely presents as new contributions. If the manuscript is positioned as a report, review, survey, expository article, lecture notes, or a similar work, do not assess results that it presents as earlier, known, surveyed, or expository material; assess only any central results that it explicitly claims as new.

A qualifying earlier paper must be a publicly available journal article, proceedings article, or stable public preprint that states and supports the result in written form. Report a result only when such a paper, publicly available before the earliest public version of the reviewed paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from a theorem in that paper.

Talks, lectures, seminar or conference presentations, slides, posters, videos, abstracts, announcements, personal communications, theses or dissertations, and other non-paper materials do not establish non-novelty, even if they state or outline the result. They may be used only to locate a qualifying earlier paper and must not be mentioned in the report.

For each non-novelty finding, provide the qualifying earlier paper, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later paper cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the reviewed paper itself explicitly presents as known, a survey item, or an alternative proof.

Novelty findings concern mathematical statements only. Do not use the novelty check to discuss author-level priority, attribution, authorship or contribution provenance, or changes in a work's author list.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, add statement-level "no prior source found" entries, or mention excluded source types or other suppressed observations even as context. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use $\$ \dots \$$ for inline mathematics and $\$ \$ \dots \$ \$$ for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.