

Correctness, proof completeness, novelty, and source audit

Diophantine approximation on abelian varieties; a conjecture of M. Waldschmidt

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Paper	arXiv:2506.19060v1
Source version	Submitted 23 June, 2025; announced June 2025.
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Prompt	Standardized prompt
Novelty cutoff	June 23, 2025
Paper license	CC BY 4.0
Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A green 'no issues found' label may coexist with yellow suspected typos or minor formal issues that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Suspected incorrect statements
02	Proofs	Suspected incomplete or incorrect proofs
03	Novelty	No novelty concerns found
04	Sources	Suspected incomplete or incorrect source use

This independent report links to the source paper on arXiv; it does not reproduce or redistribute that paper.

01 / STATEMENTS

Statements

Suspected incorrect statements

The headline equivalence with non-very-well-approximability, the general upper bound, and the rank-one elliptic-curve exponent appear mathematically correct after the explicit repairs recorded below; Theorem 2.3 is also independently supplied by the cited Moshchevitin-Neckrasov theorem. The manuscript nevertheless contains false auxiliary statements: Lemma 2.2 fails for matrices with an exact integer relation, the set declared hyperplane absolute winning on page 7 omits its denominator cutoff and can be empty, and the proof of isogeny invariance states the comparison inequality in the wrong direction. The rank-zero case of Theorem 1.5 is also not defined under the paper's matrix conventions.

Theorems 1.5 and 2.3: The Waldschmidt equivalence is correct in positive rank

PDF pages 3 and 6-8 · Theorems 1.5 and 2.3, Lemma 2.5, and Remark 2.4 · arXiv:2506.19060v1

Correct after the stated scope and proof repairs

For rank $r > 0$, Waldschmidt's Lemma 5.1 converts the height exponent to the coefficient exponent by $\widehat{h}(\sum q_j \beta_j) \asymp \|q\|^2$. With $B = J^{-1}H$, Theorem 2.3 applied with $m = g$ and $n = r$ then says exactly that the uniform error $Q^{-r/g+\varepsilon}$ holds for every shift if and only if B is not very well approximable. The finite-index passage from all rational points to $\Gamma = \sum_j \mathbb{Z}\beta_j$, and invariance under changing periods, lifts, or a full-rank rational subgroup, are standard but should be stated. The reverse proof printed in this paper is invalid; the result itself is independently established by Theorem 1(A)-(B) of arXiv:2503.21180v1.

Verifiable source: Moshchevitin and Neckrasov, exact arXiv:2503.21180v1

Lemma 2.2: The local-minimum statement fails when there is an exact relation

PDF page 5 · Lemma 2.2 and proof · arXiv:2506.19060v1

Suspected incorrect statement as written

Take $m = n = 1$, $A = 0$, and $\psi(x) = 1/x$. The matrix is ψ -approximable, because $Aq + p = 0$ for every nonzero q with $p = 0$. But $\Lambda_A = \mathbb{Z}^2$ and, for $t \geq 0$, $\|g_t \Lambda_A\| = e^{-t}$, which is strictly decreasing and has no local minima. Thus there is no unbounded sequence $\{t_k\}$ of the asserted kind. The lemma becomes plausible after excluding a nonzero lattice vector whose expanding block vanishes and treating that exact-relation case separately; the paper does neither, and later invokes the lemma in a case that includes rational very-well-approximable matrices.

Verifiable source: Exact manuscript, arXiv:2506.19060v1

Theorem 2.3 reverse direction: The displayed target set is not hyperplane absolute winning as defined

PDF page 7 · definition of $D_{c,\mu}$ and the claimed HAW strengthening · arXiv:2506.19060v1

Suspected incorrect statement as written

The definition of $D_{c,\mu}$ quantifies over all $q \in \mathbb{Z}^n$ but omits the required restriction $\|q\| < Q$. For $m = n = 1$ and an irrational Liouville number A , the orbit $A\mathbb{Z} + \mathbb{Z}$ is dense, so for every γ and every positive threshold there is an unrestricted q below that threshold. Hence the printed $D_{c,\mu}$ is empty, and therefore cannot be HAW. Restoring $\|q\| < Q$ is necessary, but the subsequent proof also needs the repairs recorded in the proofs panel.

Verifiable source: Exact manuscript, arXiv:2506.19060v1

Lemma 1.7: The comparison inequality is reversed although isogeny invariance survives

PDF page 4 · proof of Lemma 1.7 · arXiv:2506.19060v1

Suspected incorrect as written · an apparently sound repair

Let $P_2 \in A_2(\mathbb{R})_0 \setminus A_2(K)$ and choose a real preimage P_1 under the isogeny. A rational preimage would map to a rational point, so $P_1 \notin A_1(K)$. Mapping approximants to P_1 shows $\sigma(P_2) \geq \sigma(P_1)$ and hence $\sigma_{A_2, K} \geq \sigma_{A_1, K}$, not the displayed $\leq$. Applying the same corrected inequality to the dual isogeny gives the reverse inequality and restores the stated equality. Thus the lemma's conclusion is sound, but the only inequality actually stated in its proof is false.

Verifiable source: [Exact manuscript](#), [arXiv:2506.19060v1](#)

Theorem 1.5 at rank zero: The matrix characterization has an unstated positive-rank hypothesis

PDF pages 2-3 and 8 · Definition 1.3, Theorem 1.5, and its proof · arXiv:2506.19060v1

Suspected ill-defined boundary case

Definition 1.3 and Theorem 2.3 assume positive matrix dimensions, whereas the matrix in Theorem 1.5 is $g \times r$. When $r = 0$ this is a $g \times 0$ matrix, so 'not very well approximable in the sense of Definition 1.3' is not defined. Waldschmidt's inequality is elementary in rank zero because its right side is h^ε and $A(\mathbb{R})_0$ is compact. The theorem should assume $r \geq 1$ or dispose of $r = 0$ separately.

Verifiable source: [Waldschmidt](#), [Conjecture 1.1](#)

Hyperplane absolute game: The game definition incorrectly guarantees a unique outcome

PDF pages 5-6 · paragraph defining HAW immediately before Theorem 2.3 · arXiv:2506.19060v1

Suspected incorrect as written · standard repair

The rules only require $\rho_{i+1} \geq \beta \rho_i$ and nesting, so the radii need not tend to zero and $\bigcap_i B_i$ need not be a singleton. The standard hyperplane-absolute game either treats failure of the radii to tend to zero as an automatic win for Alice or formulates the target condition for the intersection. The cited source's Proposition 2.3 remains valid under the standard definition, not the literal definition printed here.

Verifiable source: [Broderick-Fishman-Kleinbock-Reich-Weiss](#), [arXiv:1106.1621](#)

Theorem 3.1: The rank-one exponent and uniform weak estimate are correct after a component repair

PDF pages 9-11 · Theorem 3.1, Lemma 3.2, and proof · arXiv:2506.19060v1

Correct after an apparently sound local repair

Replace the chosen free generator Q by $2Q$ if necessary so that it lies in $E(\mathbb{R})_0$. Its logarithm ratio θ/ω is irrational, and for a nonrational target P the shift is not in $\mathbb{Z}(\theta/\omega) + \mathbb{Z}$. Niven's nonhomogeneous Minkowski theorem gives infinitely many q with distance $< |\omega|/(4|q|)$, while $\hat{h}([q]Q) = q^2 \hat{h}(Q)$. This proves the uniform lower bound $\sigma_{E, K} \geq 1/2$; the repaired Lemma 2.6 gives the matching upper bound in rank one.

Verifiable source: [Niven](#), [Minkowski's theorem on nonhomogeneous approximation](#)

02 / PROOFS

Proofs

Suspected incomplete or incorrect proofs

The independent reverse proof of Theorem 2.3 is not valid as printed: its target set loses the height condition, Lemma 2.2 has an exact-relation counterexample, a crucial symbol is undefined, and the final set formula is malformed. Lemma 1.7 reverses its comparison; Lemma 2.6 omits the finite-index and torsion cosets and strengthens its hypothesis from nonrational to all points; and Theorem 3.1 assumes without justification that a free generator lies in the real identity component. Explicit repairs preserve the advertised headline results, and the cited Moshchevitin-Neckrasov theorem independently proves Theorem 2.3.

Theorem 2.3, forward implication: The exponent and successive-minima calculation checks out

PDF pages 6-7 · first half of the proof of Theorem 2.3 · arXiv:2506.19060v1

Correct and complete after citing standard transference

With $\theta_\sigma = (n\sigma - m)/(mn(\sigma + 1))$, direct calculation gives

$$(1/m - \theta_\sigma)/(1/n + \theta_\sigma) = 1/\sigma.$$

Therefore $t = (1/n + \theta_\sigma)^{-1} \log(Q/c_2)$ yields $\|q\| < Q$ and error $c_2^{1+1/\sigma} Q^{-1/\sigma}$; setting $\varepsilon = n/m - 1/\sigma$ gives exactly Equation (2.1). Mahler duality and comparability of covering radius with the last successive minimum justify the uniform translate estimate. The only omitted input is the standard qualitative Khintchine transference equivalence that A is VWA if and only if $A^\top$ is VWA.

Verifiable source: Kleinbock and Margulis, arXiv:math/9812088v2, Lemma 8.3 and Theorem 8.5

Theorem 2.3, reverse implication: The HAW argument loses the bounded-denominator condition

PDF pages 7-8 · reverse proof of Theorem 2.3 · arXiv:2506.19060v1

Suspected incorrect as written; theorem independently verified

After defining $D_{c,\mu}$ without $\|q\| < Q$, the proof assumes from $\gamma \notin D_{c,\mu}$ that there are q_k with $\|q_k\| < Q_k$; that inference is unavailable. The point $y_k = g_{t_k}(z_k, 0)^\top$ then uses an undefined z_k , evidently intended to be γ . With $Q_k = c \exp((\theta_\mu + 1/n)t_k)$, the first coordinate contributes $c^{1-1/\mu} e^{\theta_\mu t_k}$ rather than the displayed $ce^{\theta_\mu t_k}$ unless the constants are renormalized. Finally, the definition of D has garbled variables ($'q, m \in \mathbb{Z}', 'n| < Q'$) and the wrong existential quantifier where all bounded q are required. These are not a complete proof. The cited Theorem 1(A)-(B) of arXiv:2503.21180v1 supplies the equivalence independently.

Verifiable source: Moshchevitin and Neckrasov, Theorem 1(A)-(B), arXiv:2503.21180v1

Lemma 2.2 and rational VWA matrices: The exact-relation case is missing from the dynamical argument

PDF pages 5 and 7 · Lemma 2.2 and its use in the reverse proof · arXiv:2506.19060v1

Suspected incomplete as written

The counterexample $A = 0$ shows that Lemma 2.2 cannot produce local-minimum times when a lattice vector contracts forever. Since a rational matrix is VWA with infinite exponent, this omitted case lies inside the reverse implication's hypothesis. It can be handled separately: a rational orbit is finite modulo $\mathbb{Z}^m$, so a shift a positive distance from that orbit immediately violates the asserted uniform approximation rate. No such case split appears in the manuscript.

Verifiable source: [Exact manuscript](#), [arXiv:2506.19060v1](#)

Lemma 1.7: The proof reaches the opposite inequality from the one printed

PDF page 4 · proof of isogeny invariance · arXiv:2506.19060v1

Suspected incorrect as written · an apparently sound repair

Surjectivity of an isogeny on real identity components lets one lift each nonrational P_2 to a nonrational P_1 . The height comparison and Lipschitz bound send every κ -approximation of P_1 to one of P_2 , proving $\sigma_{A_2, K} \geq \sigma_{A_1, K}$. The manuscript writes $\leq$. Reversing that symbol and applying the dual isogeny proves equality. The canonical-height pullback can be written directly as $\hat{h}_D(\phi Q) = \hat{h}_{\phi^* D}(Q)$; this also avoids the indirect and dimension-restricted Silverman citation.

Verifiable source: [Exact manuscript](#), [arXiv:2506.19060v1](#)

Theorem 1.5 and Lemma 2.5: Finite-index and choice-invariance reductions are omitted

PDF page 8 · Lemma 2.5 and proof of Theorem 1.5 · arXiv:2506.19060v1

Suspected incomplete as written · standard repair

Waldschmidt's conjecture permits all of $A(K)$, while Lemma 2.5 uses the subgroup Γ generated by a maximal independent subset of $A(K) \cap A(\mathbb{R})_0$. This subgroup has finite index modulo torsion. Choose N with $N(A(K) \cap A(\mathbb{R})_0) \subseteq \Gamma$ and lift a target through multiplication by N ; canonical-height quadraticity and Lipschitz continuity transfer the uniform estimate. The proof should also note that changing period bases, logarithm lifts, or a full-rank Mordell-Weil subgroup changes $J^{-1}H$ by integer or rational finite-index transformations that preserve the VWA exponent. These omissions are repairable and do not alter Theorem 1.5 for $r > 0$.

Verifiable source: [Waldschmidt](#), [Lemma 5.1](#), [journal pages 40-41](#)

Lemma 2.6: The bad-shift contradiction ignores rational exceptions and finite cosets

PDF page 9 · proof of Lemma 2.6 · arXiv:2506.19060v1

Suspected incomplete as written · an apparently sound repair

From $\sigma_{A, K} > r/(2g)$ the definition gives the improved exponent only for $P \notin A(K)$, not 'for every $P \in A(\mathbb{R})_0$ ' as written. Also, a maximal independent set need not generate every rational point: torsion and finitely many free cosets remain. Let N kill this finite quotient. Choose y in the full-dimensional Bugeaud-Harrap-Kristensen-Velani set Bad_B and outside the countable set of rational lifts, and take a target with $J^{-1}\xi = y/N$. For any rational Q , multiplying its lift by N turns its coset and torsion term into a period, so the lower bound for $Bk + p + y$ gives $\text{dist}(P, Q) \gg \hat{h}(Q)^{-r/(2g)}$. This contradicts every exponent $\mu/2 > r/(2g)$ and completes the lemma, including all rational cosets.

Verifiable source: [Bugeaud-Harrap-Kristensen-Velani](#), [Theorem 1](#), [arXiv:0807.3863](#)

Theorem 3.1: The chosen generator may not have a real logarithm

PDF pages 9-11 · proof of Theorem 3.1 · arXiv:2506.19060v1

Suspected incomplete as written · an apparently sound repair

A generator of the free summand of $E(K)$ need not lie in $E(\mathbb{R})_0$, so the asserted $\theta \in \mathbb{R}$ with $\exp_E(\theta) = Q$ need not exist. Since $E(\mathbb{R})/E(\mathbb{R})_0$ has order at most two, replacing Q by $2Q$ gives an infinite-order rational point in the identity component and only changes the uniform constant. The later sentence calling the exponential map an isometry is also too strong for an arbitrary chosen Riemannian metric; the earlier bounded-distortion statement and the displayed $\asymp$ are the correct argument. With those two replacements, the Minkowski-height proof closes.

Verifiable source: [Niven 1961, exact Proceedings article](#)

03 / NOVELTY

Novelty

No novelty concerns found

No non-novelty findings.

04 / SOURCES

Sources

Suspected incomplete or incorrect source use

The central primary sources were located and their imported conclusions checked. Waldschmidt's Conjecture 1.1 and Lemma 5.1, Moshchevitin-Neckrasov Theorem 1(A)-(B), Bugeaud-Harrap-Kristensen-Velani Theorem 1, Kleinbock-Margulis Lemma 8.3 and Theorem 8.5, Broderick et al. Proposition 2.3, and Niven's nonhomogeneous theorem all support the roles assigned to them after the repairs in this audit. Source use is nevertheless incomplete: Silverman's elliptic-curve text is cited for general abelian-variety canonical heights and does not directly give the displayed pullback comparison, while the focal paper also mislabels the Kleinbock-Margulis lemma as a theorem.

Review form: The exact current arXiv source is version 1

Official 11-page PDF and source record · submitted June 23, 2025 · arXiv:2506.19060v1

Exact source verified

The arXiv API and official PDF both identify version 1, dated June 23, 2025, with no later revision. All manuscript page references in this audit use that exact 11-page version.

Verifiable source: [Official arXiv:2506.19060v1](#)

Waldschmidt [16]: The conjecture and coefficient-height equivalence match the source

Focal PDF pages 3 and 8 · source journal page 29, Conjecture 1.1, and pages 40-41, Lemma 5.1 · arXiv:2506.19060v1

Applicable and sufficient

Waldschmidt's Conjecture 1.1 is the focal Conjecture 1.4 with the same dimension, rank, height, and exponent. Lemma 5.1 states the equivalence between coefficient-bounded approximation at rate $T^{-\theta}$ and canonical-height-bounded approximation at rate $h^{-\theta/2}$ for a subgroup generated by independent rational points. This is exactly the conversion used in Lemma 2.5, up to renaming the independent scale variables and the omitted finite-index reduction.

Verifiable source: [Waldschmidt, Density measure of rational points on Abelian varieties](#)

Moshchevitin-Neckrasov [12]: The cited transference theorem independently establishes Theorem 2.3

Focal PDF pages 5 and 8 · Remark 2.4 · source Theorem 1(A)-(B), arXiv:2503.21180v1 · arXiv:2506.19060v1

Applicable and sufficient

In the source's notation, take the focal $m \times n$ matrix as Θ and $f(T) = T^{-(1+\delta)}$. The conjugate function is $g(T) = T^{-1/(1+\delta)}$. Source Theorem 1(A) makes every shift uniformly g -Dirichlet when $\Theta^\top$ is not f -approximable; after converting $T = Q^n$, the error exponent is $n/(m(1+\delta)) = n/m - \varepsilon$. Part (B) makes the good-shift set null when the transpose is sufficiently f -approximable. Varying δ is exactly the non-VWA/VWA dichotomy. Thus the source rescues the theorem, though not the focal paper's claimed independent HAW proof.

Verifiable source: [Moshchevitin and Neckrasov, arXiv:2503.21180v1](#)

Bugeaud-Harrap-Kristensen-Velani [3]: The full-dimensional bad-shift theorem is quoted correctly

Focal PDF page 9 · Lemma 2.6 · source Theorem 1, Mathematika 56 (2010), 193-202 · arXiv:2506.19060v1

Applicable after the finite-index repair

The source defines Bad_A by the uniform lower bound $\|Aq - x\| \geq c(x)\|q\|^{-m/n}$ for every nonzero integer q and proves $\dim \text{Bad}_A = n$ for every real $n \times m$ matrix. With focal dimensions $g \times r$, this is precisely the exponent r/g needed in Lemma 2.6. The source controls integer coefficient vectors; the focal proof still has to perform the omitted finite-index and torsion reduction before applying it to every rational point.

Verifiable source: Bugeaud-Harrap-Kristensen-Velani, exact arXiv:0807.3863v2

Kleinbock-Margulis [6]: The dynamical correspondence is applicable but one locator is wrong

Focal PDF pages 5-7 · Lemma 2.2 and Theorem 2.3 · source Lemma 8.3 and Theorem 8.5, Inventiones 138 (1999) · arXiv:2506.19060v1

Result appears correct · suspected stale locator

The function correspondence with Equations (8.2a)-(8.2b) is Lemma 8.3, not 'Theorem 8.3' as cited twice in the focal paper. Theorem 8.5 is the required lattice/approximation correspondence. The 2018 erratum to Kleinbock-Margulis concerns a smoothing proposition elsewhere and does not alter Lemma 8.3 or Theorem 8.5. These results support the focal forward calculation but do not imply the false exact-relation version of focal Lemma 2.2.

Verifiable source: Kleinbock and Margulis, arXiv:math/9812088v2

Broderick-Fishman-Kleinbock-Reich-Weiss [2]: The HAW closure properties are correct under the source definition

Focal PDF pages 5-6 · HAW definition and Proposition 2.3 citation · source Proposition 2.3 · arXiv:2506.19060v1

Applicable with a definition correction

Source Proposition 2.3 supplies full Hausdorff dimension and countable-intersection stability. The focal text's unique-point formula omits the standard treatment of plays whose radii do not shrink to zero, so the citation does not validate that literal formulation. Restoring the source game's convention makes the two imported properties applicable.

Verifiable source: Broderick et al., exact arXiv:1106.1621v1

Niven [13]: The nonhomogeneous $1/4$ estimate matches the rank-one use

Focal PDF pages 10-11 · proof of Theorem 3.1 · source Proceedings AMS 12 (1961), 992-993 · arXiv:2506.19060v1

Applicable and sufficient

Niven's theorem gives infinitely many integer pairs with $|q||q\alpha + p - \beta| < 1/4$ when α is irrational and $\beta \notin \mathbb{Z}\alpha + \mathbb{Z}$. Lemma 3.2 verifies exactly those hypotheses for the logarithms of the generator and target. Together with quadraticity of the canonical height, it yields the claimed $\hat{h}^{-1/2}$ rate.

Verifiable source: Niven, exact Proceedings AMS record

Silverman [15]: The cited elliptic-curve text does not directly support the general abelian formulas

Focal PDF pages 3-4 and 9 · canonical-height definition, Lemma 1.7, and Section 3 setup · Silverman Chapters VIII.5 and VIII.9 · arXiv:2506.19060v1

Suspected incorrect or insufficient source use

Silverman's book treats elliptic curves, whereas pages 3-4 of the focal paper invoke it for canonical heights and isogenies of arbitrary abelian varieties. Theorem VIII.5.6 is a Weil-height functoriality estimate for morphisms and is not the displayed canonical-height comparison with $\widehat{h}_{\phi^*D}$. The needed general fact is standard and stronger: canonical heights satisfy $\widehat{h}_D(\phi Q) = \widehat{h}_{\phi^*D}(Q)$. The citation can be repaired by using a general abelian-variety height source, such as the already cited Lang chapter, and by separating the Riemannian metric from the choice of ample symmetric divisor.

Verifiable source: Silverman, *The Arithmetic of Elliptic Curves*, official author page

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Suspected typographical errors and minor formal issues remain fully visible in yellow with their locations and proposed repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs result from green (no issues found).

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. [Official arXiv:2506.19060v1](#)
2. [Moshchevitin and Neckrasov, arXiv:2503.21180v1](#)
3. [Exact manuscript, arXiv:2506.19060v1](#)
4. [Waldschmidt, Density measure of rational points on Abelian varieties](#)
5. [Broderick et al., exact arXiv:1106.1621v1](#)
6. [Niven, exact Proceedings AMS record](#)
7. [Kleinbock and Margulis, arXiv:math/9812088v2](#)
8. [Bugeaud-Harrap-Kristensen-Velani, exact arXiv:0807.3863v2](#)
9. [Silverman, The Arithmetic of Elliptic Curves, official author page](#)

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Do not investigate, infer, or comment on authorship, contribution credit, acknowledgments, changes in author lists between versions or sources, collaboration history, or research conduct. These matters are outside the audit and must not appear in any finding, summary, source assessment, or report text. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only comparison of central mathematical results with earlier papers. First identify only the central results that the reviewed paper genuinely presents as new contributions. If the manuscript is positioned as a report, review, survey, expository article, lecture notes, or a similar work, do not assess results that it presents as earlier, known, surveyed, or expository material; assess only any central results that it explicitly claims as new.

A qualifying earlier paper must be a publicly available journal article, proceedings article, or stable public preprint that states and supports the result in written form. Report a result only when such a paper, publicly available before the earliest public version of the reviewed paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from a theorem in that paper.

Talks, lectures, seminar or conference presentations, slides, posters, videos, abstracts, announcements, personal communications, theses or dissertations, and other non-paper materials do not establish non-novelty, even if they state or outline the result. They may be used only to locate a qualifying earlier paper and must not be mentioned in the report.

For each non-novelty finding, provide the qualifying earlier paper, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later paper cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the reviewed paper itself explicitly presents as known, a survey item, or an alternative proof.

Novelty findings concern mathematical statements only. Do not use the novelty check to discuss author-level priority, attribution, authorship or contribution provenance, or changes in a work's author list.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, add statement-level "no prior source found" entries, or mention excluded source types or other suppressed observations even as context. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use $\$ \dots \$$ for inline mathematics and $\$ \$ \dots \$ \$$ for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- *Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.*
- *Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.*
- *Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.*

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Authorship, contribution credit, acknowledgments, author-list changes, collaboration history, and authorship or contribution provenance remain outside scope and must not appear in the Sources assessment. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.