MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Bad approximability, bounded ratios and Diophantine exponents

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Prompt	Standardized prompt
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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

The norm-invariance theorem, the qualitative implication and counterexample results, the realizability theorems for ordinary and uniform exponents, and the stated upper bounds other than Theorem 2.8(2) are supported. No counterexample was found to Theorem 2.8(2), but its proof controls only infinitely many indices and therefore does not establish the required limsup bound. Theorem 2.10(2) also needs the local boundary correction $n \geq 2$ (or the explicit hypothesis (2.6)).

Theorems 2.1–2.4: Norm invariance, logical implications, counterexamples, and the three-dimensional nonsingularity result

Pages 4–5; proofs in Sections 4.1–4.2 and 5.1 · arXiv:2505.15964v3

Correct

Equivalence of norms and the bounded-distance description of successive-minima functions preserve properties (B) and (C). The implications in Theorem 2.2 and the $m + n = 3$ nonsingularity argument follow with the stated hypotheses. For Theorem 2.3, the constructed connected templates have the required extremal-slope behavior; sparse excursions destroy exactly the selected property while their zero-density contribution leaves both contraction averages equal to mn, giving the asserted full Hausdorff dimension through the cited variational principle.

Verifiable source: Paper, version 3

Theorems 2.6 and 2.9: Realizable ordinary and uniform exponents

Pages 5–6 and 23–33 · Sections 5.2–5.4 · arXiv:2505.15964v3

Correct

After the mechanically determined index correction recorded in Part 2, the families $\mathcal{P}_{v,w}$ tile the complete interval of admissible values of $\liminf P_1(q)/q$ while enforcing exactly the required combinations of (B), (C), and failure of (A). The families $\mathcal{Q}_v$ and $\mathcal{R}_u$ similarly cover the displayed intervals for $\limsup P_1(q)/q$. Formula (3.12), connectedness, condition U, and the variational principle then yield totally irrational matrices with each claimed exponent and property combination. The edge case $n = m(m-1)$ is handled separately immediately before the indexed construction.

Verifiable source: Paper, version 3

Theorem 2.8(1) and (3): Upper bounds for the ordinary exponent outside the case $m = 2$

Pages 17 and 19 · Section 4.3.1 · arXiv:2505.15964v3

Correct

For $m = 1$, the covolume relation $L_{i-1}X_i \asymp 1$ together with (C) gives $L_iX_i \asymp 1$ and hence $\omega(\xi) \leq 1$. For $m \geq 3$, the quoted fixed-step growth $X_{k+3m+n-1} \geq 2X_k$ makes X_k exponentially large in k, whereas (C) makes $-\log L_k$ at most linear in k; their ratio is therefore uniformly bounded, proving $\omega(\xi) < \infty$.

Verifiable source: Paper, version 3

Theorem 2.8(2): The claimed bound $\omega(\xi) \leq 2$ is not established

Pages 17–18 · proof of Theorem 2.8, Statement 2 · arXiv:2505.15964v3

Not able to verify

In the nonplanar case, determinant estimates prove $L_{i+1}X_{i+1}^2 \gg 1$ (or the stronger $L_{i+1}^2 X_{i+1} \gg 1$) only at the infinitely many indices for which x_{i-1}, x_i, x_{i+1} are independent. But

$$\omega(\xi) = \limsup_{j\to\infty} \frac{-\log L_j}{\log X_j}$$

requires control at every sufficiently large index, not merely along one infinite subsequence. Property (C) bounds successive ratios of the L_j but supplies no bound on the intervening ratios X_{j+1}/X_j, so immediate-consequence closure does not bridge those blocks. No later result repairs this: Lemma 4.2 assumes both (B) and (C), whereas this theorem assumes only (C). No counterexample to the statement was found, so the supported outcome is Not able to verify, not Incorrect.

Verifiable source: Paper, version 3

Theorem 2.10: Uniform-exponent upper bounds, with one boundary correction

Pages 6–7 and 19–22 · Theorem 2.10 and Section 4.3.2–4.3.3 · arXiv:2505.15964v3

Minor formal correction

Statement 1 is supported. Statement 2 is supported for $n \geq 2$, when total irrationality implies condition (2.6), and more generally whenever (2.6) is assumed. As printed, however, the theorem starts with $n \geq 1$ and says that a totally irrational $\xi \in \mathbb{R}^{n\times 2}$ therefore satisfies (2.6). For $n = 1$, the ambient space $\mathbb{R}^3$ is itself a three-dimensional rational subspace, so (2.6) is impossible; totally irrational badly approximable linear forms satisfy (B) and (C) and have $\widehat{\omega} = 2$, as Remark 2.11 itself notes. Replace $n \geq 1$ by $n \geq 2$ in Statement 2, or explicitly add (2.6). This removes only the identified boundary case, requires no new argument, and affects no later application.

Verifiable source: Paper, version 3

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proof of Theorem 2.8(2) has a substantive unresolved gap: an estimate on infinitely many independent triples does not control the limsup defining the ordinary exponent. The other central proof chains are verifiable after the localized typographical and boundary corrections listed below; those corrections do not lower the overall status independently.

Theorem 2.8(2): The independent-triple subsequence does not control all best approximants

Page 18 · two cases in the proof of Statement 2 · arXiv:2505.15964v3

Incomplete as written

The planar alternative is complete and even yields $\omega(\xi) = 1$. In the other alternative, the proof obtains a lower bound for L_{i+1} only whenever three successive best-approximation vectors are independent, then concludes from the infinitude of those indices that $\omega(\xi) \leq 2$. A limsup upper bound instead requires a corresponding estimate throughout every intervening dependent block. Condition (C) controls L_i/L_{i+1}, but without (B) it does not control the growth of X_i across such a block. Repair classification: No repair supplied; a new block estimate under (C) alone, or another global argument, is required.

Verifiable source: Paper, version 3

Theorem 2.10(2): Pattern argument and the $4/3$ bound

Pages 20–22 and Appendix A · proof of Theorem 2.10(2) · arXiv:2505.15964v3

Correct and complete

Under condition (2.6) and after the local corrections listed below, successive independent triples delimit a block lying in one two-dimensional subspace. Lemma 4.2 and (B) give $L_\nu \ll X_\nu^{-1} X_k^{1-\alpha}$, while an independent-triple determinant gives $L_\nu \gg X_\nu^{-2}$ and the four-vector determinant gives $1 \ll L_{\nu-1} L_{k-1} X_k X_{k+1}$. Thus

$$X_k^{\alpha-1} \ll X_\nu \ll X_k^{3-2\alpha},$$

forcing $\alpha \leq 4/3$. Since this holds for every $1 < \alpha < \widehat{\omega}(\xi)$, the claimed bound follows.

Verifiable source: Paper, version 3

Theorem 2.8(2) · proportional-coordinate determinant: The second error row uses the wrong coefficients

Page 18 · final determinant in the proportional case · arXiv:2505.15964v3

Typo

In the row beginning with $y_{l,i-1}$, the subtracted coefficients are printed as $\xi_{j,1}, \xi_{j,2}$. They must be $\xi_{l,1}, \xi_{l,2}$ in all three entries. The row label l and the definition of L_i determine this correction uniquely; with it both lower rows are $O(L)$ and the displayed determinant bound follows unchanged.

Section 4.3.3 · vector labels: Two local notation errors have unique corrections

Pages 20–21 · opening paragraph and Equation (4.7) · arXiv:2505.15964v3

Typo

The matrix ξ is called $2 \times n$ although the paper's convention and every displayed vector require $\xi \in \mathbb{R}^{n\times 2}$. Equation (4.7) then repeats x_{k-1} in a purported list of four independent vectors; the immediately following rank-four matrix shows that the list must be

$$x_{\nu-1},\ x_{k-1},\ x_k,\ x_{k+1}.$$

Both replacements are mechanically determined and the subsequent calculation already uses the corrected objects.

Section 5.2 · template parameters: A sign omission and an exponent-index shift need correction

Pages 26–29 · Remark 5.4 and proofs of Theorem 2.6(1),(3) · arXiv:2505.15964v3

Typo

Remark 5.4 prints $\lambda_k \geq \log q_k$, impossible because $\lambda_k \leq 0$; the singularity condition is $-\lambda_k q_k \geq \log q_k$ (equivalently $\lambda_k q_k \leq -\log q_k$). Later, for $n > m(m-1)$, the printed choice $m^{\ell_n - 1} - m < n \leq m^{\ell_n} - m$ makes $v_{\ell_n} = m^{\ell_n} - m \geq n$, contrary to the required $v < n$. The uniquely consistent shift is

$$m^{\ell_n} - m < n \leq m^{\ell_n + 1} - m.$$

It makes every v_ℓ admissible and makes intervals (5.7)–(5.9) tile $[-n, 0]$; the equality $n = m(m-1)$ was already handled in the preceding paragraph. Apply the same shift in Section 5.2.6.

Section 5.3 · interval notation: The rising segment has the wrong right endpoint

Page 29 · first paragraph of Section 5.3 · arXiv:2505.15964v3

Typo

After choosing $q_k < p_k < q_{k+1}$, the text says that Q_1 has positive or constant slope on $[p_k, q_k]$, a reversed interval. The constructions immediately below determine the intended segment as $[p_k, q_{k+1}]$. This correction changes no template or exponent calculation.

Appendix A · ellipse definition: The error threshold must be L_l

Page 34 · definition of G_l in the proof of Lemma A.1 · arXiv:2505.15964v3

Typo

The ellipse is printed as

$$G_l = \pi \cap \{x : \|\xi x - y\|_2 \leq L_{l+1}\},$$

but the next lines require both $z_l, z_{l+1} \in G_l$ and explicitly use $z_l \in G_l$. Since the errors decrease strictly, z_l need not satisfy the printed threshold. Replace L_{l+1} by L_l. This is uniquely forced by those inclusions and makes the ensuing area and covolume argument valid without changing its bounds.

Appendix A · index range: One endpoint is short by one index

Pages 33–34 · Case 2 in the proof of Lemma A.1 · arXiv:2505.15964v3

Minor formal correction

With $k_1 = k - C_2$, the hypothesis $k \geq \nu + 2C_2$ yields only $k_1 \geq \nu + C_2$. Applying (A.5) with starting index $\nu + 1$ to conclude $X_{k_1} \geq Z_{\nu+1}$ requires $k_1 \geq \nu + 1 + C_2$. Increase the Case 2 threshold to $k \geq \nu + 2C_2 + 1$ and absorb the remaining single index into Case 1 (or replace k_1 by $k - C_2 + 1$). The finite-step comparison (A.4) already covers this local adjustment, so no later estimate changes.

Remaining central proofs: Best-approximation and template proof chains

Sections 4.1–4.2 and 5.1–5.4 · arXiv:2505.15964v3

Correct and complete

The bounded-distance minima lemma is applied with the correct norm-equivalence constants; the low-dimensional covolume arguments preserve total irrationality and all cases; and the template constructions satisfy the slope, ordering, summation, connectedness, and extremal-slope conditions needed by the variational principle. Sparse exceptional intervals contribute zero asymptotic density, while the exponent formulas use the correct liminf or limsup in each construction. No further nontrivial omitted obligation was identified.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2505.15964v3
2. Paper, version 3

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.