

Correctness, proof completeness, novelty, and source audit

On Lagrangianity of p -supports of holonomic D-modules and q -D-modules

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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings
04	Sources	Contains incorrect or incomplete source use

This independent report links to the source paper on arXiv; it does not reproduce or redistribute that paper.

01 / STATEMENTS

Statements

Contains unsupported statements

The characteristic-zero Bitoun theorem, its smooth-variety corollary, and the q -analog of Bernstein's theorem are correct after the verified local proof repair and literal notation corrections below. The q -support theorems are also established when the residue characteristic is zero or exceeds the rank of the transformed bundle. Their stated fixed-positive-characteristic scope is not verified when that rank is at least the characteristic. Independently, the cluster theorem omits the full-order condition needed for quantum mutations to identify the centers of adjacent charts at roots of unity: for $N = \text{ord}(q)$ one needs $\gcd(N, d_k) = 1$. Both limitations propagate to Theorem 3.14.

Theorem 2.9 and Corollary 2.11: The characteristic-zero p -support conclusions are correct

PDF pages 5-6 · Theorem 2.9 and Corollary 2.11 · arXiv:2504.19329v4

Correct

After spreading the symplectic transformation of Theorem 2.8 over a localization of the finitely generated coefficient ring, reduction gives a symplectic automorphism of the characteristic- p Weyl algebra and hence of its center; it preserves Lagrangianity. The transformed module is a vector bundle with a flat connection of a fixed rank n . Outside the finitely many residue characteristics $p \leq n$, each reduction is liftable and Corollary 2.6 applies. For the smooth-variety statement, the transfer-module calculation for a closed embedding gives exactly the displayed cotangent correspondence for p -supports; its local verification and source locator are recorded below. The false sentence in the proof of Lemma 2.3 has the verified replacement recorded in the Proofs dimension, so it does not change these theorem outcomes.

Theorem 3.3: The q -analog of Bernstein's theorem is established

PDF pages 8-10 · Lemma 3.1 through Theorem 3.3 · arXiv:2504.19329v4

Correct

The orbit lemma patches local freeness over the Laurent polynomial algebra, while Lemmas 3.4-3.6 reduce one noncommuting variable at a time after an integral symplectic change of coordinates. The Zariski density of $\text{Sp}(2s, \mathbb{Z})$ in $\text{Sp}(2s, \mathbb{C})$ supplies the integral point in the nonempty open set used in Lemma 3.4. Bernstein's inequality excludes the free-submodule alternative for a nonzero holonomic module, completing the induction. The index and product-order errors listed in the Proofs dimension are uniquely repairable and do not affect the result.

Section 3.7: The dimension- r definition is consistent after forcing finite Casimir support

PDF pages 14-15 · paragraph preceding Theorem 3.13 · arXiv:2504.19329v4

Correct in the supported rank range

When the skew form has rank $2r$ on an s -torus, a Lagrangian inside the whole Poisson variety would ordinarily have dimension $s - r$, not r . Here the stronger holonomicity bound forces the missing $s - 2r$ Casimir directions to be zero-dimensional. After the finite isogeny used in the manuscript, the algebra is finite over the tensor product of a rank- $2r$ quantum torus and a central Laurent algebra C in $s - 2r$ variables. If the C -support has dimension d , Bernstein's inequality on the nondegenerate factor gives $\text{GKdim } M \geq r + d$. Thus $\text{GKdim } M \leq r$ forces $d = 0$: a finite-length module is supported on finitely many Casimir fibers. On each $2r$ -dimensional symplectic leaf, a coisotropic support of total dimension at most r must have dimension exactly r and is Lagrangian. The wording is compressed, but the apparent dimension mismatch is therefore resolved. This does not repair the separate fixed-positive-characteristic rank gap inherited from Theorem 3.11.

Theorems 3.11, 3.13, and 3.14: The fixed-positive-characteristic rank range and non-admissible cluster orders are unsupported

PDF pages 13-16 · Theorems 3.11, 3.13, 3.14 and their reductions to Proposition 3.9 · arXiv:2504.19329v4

Not able to verify in the two stated regimes

Theorem 3.11 is stated over an algebraically closed field of any characteristic. Its proof transforms M into a free Laurent-polynomial module of some fixed rank n and then invokes Proposition 3.9. In residue characteristic $p > 0$, that proposition explicitly assumes $n < p$: its trace argument divides by joint-eigenspace multiplicities $m_k \leq n$. Over a characteristic-zero base, one may discard the finitely many residue characteristics $p \leq n$, so the argument works. Over a base field of fixed characteristic p , every specialization has the same characteristic, and when $n \geq p$ no generic-open restriction makes the hypothesis true. Example 3.10 shows that the rank restriction cannot simply be erased from Proposition 3.9, although it does not provide a fixed generic- q counterexample to Theorem 3.11. Thus this regime is not established, and Theorems 3.13-3.14 inherit the limitation. Theorem 3.14 has a second, independent scope failure. If $\tilde{B}^T \Lambda = (D \mid 0)$, the two quantum exchange summands in direction k commute with factor q^{d_k} . At a torsion value with $N = \text{ord}(q)$, the classical mutation formula for their N th powers requires $\text{ord}(q^{d_k}) = N$, equivalently $\gcd(N, d_k) = 1$. Otherwise quantum-binomial cross terms can survive, so the centers of adjacent quantum charts do not glue by the displayed classical mutation. Parity is not an intrinsic extra condition for Etingof's parameter q ; MNTY's half-parameter Frobenius convention imposes odd half-parameter order as a sufficient additional hypothesis. Whenever some relevant $d_k > 1$, which the theorem allows, the characteristic-zero torsion orders divisible by a prime factor of that d_k form an infinite, hence Zariski-dense, subset of $\mathbb{G}_m$ and cannot be removed by the asserted generic Zariski-open restriction. If every relevant $d_k = 1$, this particular obstruction disappears. A repair in the general stated scope is verified only after restricting to an explicitly full-order torsion family.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The q-support proof omits the entire fixed-characteristic range in which the transformed rank is at least the characteristic, and the cluster proof additionally omits the root-of-unity admissibility needed for adjacent chart centers to mutate classically. Separately, Lemma 2.3 uses a false statement about the ideal pS , but a one-line local-ring argument gives a verified repair. The spread-out symplectic reduction in Theorem 2.9 is recoverable, and chartwise supports glue after explicitly restricting to admissible torsion orders. The unnormalized quantum-torus monomial transformations compose only projectively, but every use has a verified normalization-free repair. Nine literal rank, exponent, locator, and algebra-symbol errors have unique harmless corrections.

Proof of Lemma 2.3: The claim that pS surjects onto the residue field is false

PDF page 3 · final sentence of the proof of Lemma 2.3 · arXiv:2504.19329v4

Incorrect as written · verified repair

The proof obtains

$$p(j\tilde{a}_{pi-1,pj} - i\tilde{b}_{pi,pj-1}) = 0$$

in the local Artinian ring S and then says that pS surjects onto the residue field k . In fact p lies in the maximal ideal because k has characteristic p , so the natural image of pS in k is zero. The needed coefficient identity nevertheless follows: if $pc = 0$ and c were a unit, then $p = 0$ in S , contrary to liftability. Hence c is a nonunit, therefore belongs to the maximal ideal, and its reduction in k is zero. Applying this to the parenthesized coefficient proves the displayed equality and the closedness of the curvature form. Repair classification: Verified repair.

Proofs of Theorems 3.11, 3.13, and 3.14: Proposition 3.9 is applied outside its rank hypothesis

PDF pages 13-16 · proofs of Theorems 3.11 and 3.14 and the paragraph preceding Theorem 3.13 · arXiv:2504.19329v4

Incomplete as written · no repair supplied in fixed characteristic $p \leq n$ for rank $n \geq p$

After Theorem 3.3, the transformed module is free of rank n over the Laurent polynomial algebra. Its reductions are liftable q-connections, but Proposition 3.9 proves Lagrangianity in positive characteristic only when $n < p$. The sentence 'the result follows from Proposition 3.9' does not address $n \geq p$ when the base field already has characteristic p . The proof of Proposition 3.9 uses $m_k \neq 0$ in the residue field for every joint-eigenspace multiplicity; when a multiplicity is divisible by p , equation (3.2) yields no logarithmic-derivative identity. The quantum-torus reduction and the cluster theorem call the same result, so they do not restore the missing case. Repair classification: No repair supplied for this regime.

Compressed reduction in Theorem 2.9: The omitted spread-out symplectic twist is formally recoverable

PDF page 5 · proof of Theorem 2.9 · arXiv:2504.19329v4

Correct and complete after immediate-consequence closure

Adjoining the finitely many coefficients of the symplectic matrix and localizing the model spreads the twist and the finite locally free connection simultaneously. Reduction of the twist is an automorphism of the Weyl algebra whose induced map on the center is symplectic, so it transports the p -support and preserves Lagrangianity. This compression creates no additional proof obligation beyond the stated source inputs.

Proof of Theorem 3.14: Chartwise support gluing fails without root-order admissibility

PDF pages 15-16 · definition of cluster p -support and proof of Theorem 3.14 · arXiv:2504.19329v4

Incomplete as written · verified only on an admissible torsion subfamily

The sentence that the central supports 'glue together' assumes that specialization of a quantum mutation induces the classical mutation between the spectra of the chart centers. For a compatible pair $\tilde{B}^T \Lambda = (D \mid 0)$, its two exchange summands A, B satisfy $AB = q^{d_k} BA$. With $N = \text{ord}(q)$, the identity $(A + B)^N = A^N + B^N$ holds in the required form only when q^{d_k} still has order N , equivalently $\gcd(N, d_k) = 1$. If q^{d_k} has smaller order, quantum-binomial cross terms survive; for example when its order is $N/2$, the middle term in $(A^{N/2} + B^{N/2})^2$ remains. Even N is not by itself an obstruction for this parameter when the full-order condition holds. MNTY's oddness condition belongs to its half-parameter Frobenius convention and is a sufficient additional hypothesis there. If some relevant $d_k > 1$, orders divisible by a prime factor of d_k give a Zariski-dense characteristic-zero torsion family, so no Zariski-open 'generic torsion' set removes them; when all relevant $d_k = 1$, this obstruction vanishes. Once one explicitly restricts to full-order specializations in the general case, localization of the same Serre-quotient object identifies the central supports on overlaps, a finite chart cover gives a common good-specialization condition, and Lagrangianity glues. That verifies a restricted theorem but not the stated one. Repair classification: Verified only after a substantive full-order restriction.

Proof of Corollary 2.11: The closed-embedding support identity follows from the transfer module

PDF page 6 · proof of Corollary 2.11 · arXiv:2504.19329v4

Correct after a verified local calculation

The equality for the p -support of $i_* M$ is asserted rather than derived in the focal manuscript, so it is proof-critical. It is exactly Bitoun's Proposition 2.4.1. It can also be checked independently in local coordinates in which the smooth closed subvariety is cut out by normal coordinates $y_1, \dots, y_c$. The transfer module identifies $i_* M$ locally with $M \otimes k[\partial_{y_1}, \dots, \partial_{y_c}]$. In characteristic p , each central function y_j^p acts by zero, the tangential central operators have the original support, and the normal operators $\partial_{y_j}^p$ are unrestricted. Thus the central support is precisely the inverse image of the original support under cotangent restriction, followed by the inclusion over $X^{(1)}$. Base change of the transfer module is compatible with reduction, and the cotangent correspondence preserves vanishing of the symplectic form and the dimension shift. This discharges the omitted calculation.

Symplectic monomial changes in Section 3.3: The cited transformations are projective rather than a literal unnormalized group action

PDF pages 8-10 · paragraph before Theorem 3.3 and its proof · arXiv:2504.19329v4

Incorrect as written · verified repair

The manuscript says that $\text{Sp}(2r, \mathbb{Z})$ acts on the quantum torus by automorphisms. Sabbah constructs an automorphism for each integral symplectic matrix, but with the displayed unnormalized monomial lifts, compositions agree only up to powers of q ; this is a projective action, or an action after choosing a compatible normalization or central extension. The proof needs only the existence of an automorphism realizing each exponent-lattice transformation. The scalar q -factors do not change finiteness, Gelfand-Kirillov dimension, or monomial exponent separation, and consecutive twists may be replaced by the single automorphism attached to the product matrix. Thus the literal group-action claim is overstated but all uses have a verified repair. Repair classification: Verified repair.

Section 2 notation: Four rank, tensor, and cross-reference symbols need unique corrections

PDF pages 2, 4-5 · Section 2.1, Lemma 2.5, and proof of Theorem 2.8 · arXiv:2504.19329v4

Typo

For a rank- n connection, M_{∇} is generated by $f_1, \dots, f_n$, not $f_1, \dots, f_r$. In the displayed exterior-power formula in Lemma 2.5, the final factor is $1^{\otimes(n-j)}$, not $1^{\otimes(n-j-1)}$; the printed exponent is even -1 when $j = n$. The proof of Theorem 2.8 must choose g by Lemma 2.7(ii), not nonexistent part (ii) of Lemma 2.3. Finally, its concluding finite-generation sentence concerns gM , not the original symbol M . Each correction is forced by the definitions and the surrounding proof.

Section 3 notation: Five q-difference and cluster symbols need unique corrections

PDF pages 9-11 and 14-15 · proofs of Lemma 3.4 and Theorem 3.3, Section 3.4, Theorem 3.13, and the quantum-cluster paragraph · arXiv:2504.19329v4

Typo

In Lemma 3.4, the recurrence coefficients belong to $\mathcal{D}_{s-1,q}(R[x_s^{\pm 1}])$, not $\mathcal{D}_{r-1,q}(R[x_r^{\pm 1}])$. At the end of the induction for Theorem 3.3, the product carrying M to the already displayed module $g_2 g_1 M$ is $g_2 g_1$, not $g_1 g_2$. In Section 3.4, a rank- n connection acts on $\mathbf{k}(x_1, \dots, x_r)^n$, not the power indexed by the root order N . Theorem 3.13 must say a holonomic $\mathcal{D}_{\Lambda,q}$ -module, not $\mathcal{D}_{r,q}$; otherwise it merely repeats Theorem 3.11 and cannot imply Theorem 3.14. In the quantum-cluster paragraph, $U_q(\Lambda, \tilde{B})$ is a flat deformation of the classical algebra $U(\Lambda, \tilde{B})$, not of itself. These are uniquely determined notation repairs.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

04 / SOURCES

Sources

Contains incorrect or incomplete source use

The proof-critical Sabbah, Bitoun, Lam, Berenstein-Zelevinsky, Matherne-Muller, and Muller-Nguyen-Trampel-Yakimov inputs were traced to exact sources. Their portions actually needed by the focal argument are supported after explicit repairs: Sabbah's monomial transformations are projective before normalization; Bitoun supplies the closed-immersion formula; Berenstein-Zelevinsky Appendix Proposition A.2(2) is false but the separately used intersection statement A.2(3) has a direct proof; and the public Matherne-Muller fallback has a gcd/lcm and exponent error in its Hartogs lemma but a verified correction establishes the codimension-two implication used by MNTY Proposition 4.6. MNTY Theorem A and Proposition 4.6 therefore remain supported, and their later false Azumaya-locus result is unrelated to the focal use. However, the focal cluster theorem omits the condition $\text{ord}(q^{d_k}) = \text{ord}(q)$ required when applying quantum mutation to root-of-unity chart centers. MNTY's half-parameter formalism additionally assumes odd half-parameter order. This is an incomplete use of the quantum-cluster sources and cannot be repaired by a generic-open torsion restriction. Losev's disclosed error remains unrelated to the exact material cited.

Sabbah, Sections 1-2: The q -difference structural inputs are applicable

Target PDF pages 8-10 · Sections 3.1-3.3 · arXiv:2504.19329v4

Applicable and sufficient

Sabbah's Proposition 1.2.1 gives simplicity of the quantum torus when q is not a root of unity, Section 1.3 gives an automorphism for each integral symplectic coordinate change, Corollary 2.1.2 gives Bernstein's inequality, Corollary 2.5.2 gives finite-dimensional generic localization, and Theorem 2.7.1 relates rational q -connections to holonomic modules. Two qualifications matter. First, the unnormalized monomial automorphisms in Section 1.3 compose only up to powers of q , so the focal phrase 'the group acts' requires normalization or projective-cocycle bookkeeping; existence of each twist is all the proof uses. Second, Theorem 2.7.1(2) has a compressed statement, but its proof chooses a finite lattice M' whose localization is the prescribed rational connection, exactly the existence required by the focal converse. The cited source is written in characteristic zero, but the proof-critical algebraic steps used here depend on q having infinite multiplicative order and on the quantum-torus division and homological arguments, not on characteristic-zero differentiation. They remain valid when q is not a root of unity in positive characteristic. The later rank obstruction comes from the focal Proposition 3.9, not from an overextension of Sabbah's theorem.

Verifiable source: C. Sabbah, *Systèmes holonomes d'équations aux q -différences*, author PDF

AGM and prior main-theorem attributions: The cited claims are independently established in the focal paper

Target PDF pages 4-6 · Lemma 2.7 through Corollary 2.11 · arXiv:2504.19329v4

Correct attribution; not an unverified dependency

The focal Lemma 2.7 supplies the complete incidence-dimension proof of the Lagrangian-projection lemma attributed to Bernstein, and Theorem 2.8 then derives the finite projection and finite module generation directly. Thus AGM Lemma 3.15 and Corollary 3.16 are historical locators rather than omitted proof inputs. Bitoun and van den Bergh are also cited for the earlier main p -support theorem and its generalization, while the focal manuscript proves its affine theorem. This finding concerns only those prior main-theorem attributions; the separate closed-immersion support formula used in Corollary 2.11 is proof-critical and is audited in the next finding through Bitoun's Proposition 2.4.1 and a local calculation.

Verifiable source: Aizenbud-Gourevitch-Minchenko, exact final arXiv version 2

Bitoun, Proposition 2.4.1: The closed-embedding formula used in Corollary 2.11 is the cited result

Target PDF page 6 · proof of Corollary 2.11 · source Section 2.4, Proposition 2.4.1 · arXiv:2504.19329v4

Applicable and sufficient

Bitoun's exact Inventiones version, Proposition 2.4.1 on printed pages 787-788, states that direct image along a closed immersion commutes with base change and gives

$$p\text{-supp}(H^0 f_{s,+} M_s) = (f'_s)_\pi \circ (f'_s)_d^{-1} (p\text{-supp}(M_s)).$$

This is the focal identity written with π and ι . Bitoun derives it from the classical transfer-bimodule description, ultimately citing Borel et al., Algebraic D-Modules, equation (4) on page 259; the local normal-coordinate computation is reproduced in the Proofs dimension, so no unexamined source step remains. Bitoun's Appendix Lemma A.0.6 on printed page 817 supplies the corresponding symplectic-form comparison. Van den Bergh is cited alongside Bitoun for the general reduction, but the exact formula and sufficient locator are in Bitoun.

Verifiable source: T. Bitoun, On the p-supports of a holonomic D-module, *Inventiones Mathematicae* 215 (2019), exact VOR

Berenstein-Zelevinsky, Appendix Proposition A.2: A false ideal statement has a local repair for the only used intersection consequence

Target PDF pages 14-16 · Section 3.8 and proof of Theorem 3.14 · arXiv:2504.19329v4

Source contains a false statement · used consequence repaired

In the exact Advances in Mathematics version, Appendix Proposition A.2(2), printed pages 453-454, claims that every two-sided ideal I of an integral based quantum torus T is generated by $I \cap Z(T)$. This is false. With $t = q^{1/2}$ and $T = \mathbb{Z}[t^{\pm 1}] \langle X^{\pm 1}, Y^{\pm 1} \rangle / (XY - t^2 YX)$, the proper two-sided ideal $I = (t - 1, X - 1)$ satisfies $I \cap Z(T) = (t - 1)$ but is strictly larger than $(t - 1)T$. The proof's assertion that every two-sided ideal is graded fails when eigencharacters coalesce after $t = 1$. The downstream Laurent argument uses only Proposition A.2(3): for relatively prime central elements z_1, z_2 , one has $Tz_1 \cap Tz_2 = Tz_1 z_2$. This follows directly by choosing coset-monomial representatives that make T free over its center and comparing coefficients. Thus Theorem 5.1 and Corollary 5.2, together with the BFZ commutative input, remain supported despite the false broader appendix claim.

Verifiable source: Berenstein-Zelevinsky, Quantum cluster algebras, exact Advances in Mathematics version

Matherne-Muller and MNTY Proposition 4.6: The codimension-two chart input survives a verified fallback repair

Target PDF pages 15-16 · cluster-locus paragraph · MNTY Proposition 4.6 · arXiv:2504.19329v4

Applicable after verified repair of the public fallback

MNTY Proposition 4.6 uses the algebraic Hartogs implication in Matherne-Muller Lemmas 4.3.1 and 4.4.2. The exact IMRN version was inaccessible; the only public arXiv version 1/author manuscript has two errors in the final converse of Lemma 4.3.1 on page 9. If $v_i = \nu(a_i) > 0$ and $\nu(a_1) = 1$, its d must be $\text{lcm}(v_i)$, not the gcd , and the displayed a_k^d must be a_k . With

$$x = a_1^{-d} \prod_{i>1} g_i^{d/v_i},$$

the identity $a_k = (f_k/g_k) a_1^{v_k}$ gives

$$x = (f_k/a_k)^{d/v_k} \prod_{i>1, i \neq k} g_i^{d/v_i} \in S[a_k^{-1}],$$

while $\nu(x) = -d$, proving the required codimension-one converse. BFZ supplies the remaining upper-bound equality, so MNTY Proposition 4.6 and Theorem A remain valid for the focal use. The false upper Azumaya-locus inclusion later in MNTY Theorem 6.1 is independent and is not used here.

Verifiable source: Matherne-Muller, Computing upper cluster algebras, public arXiv version 1 fallback

Root-of-unity mutation scope: The cluster-support gluing omits the admissible-order hypothesis

Target PDF pages 15-16 · definition of cluster p -support and Theorem 3.14 · arXiv:2504.19329v4

Incomplete source use

The generic- q quantum Laurent theorem identifies the fraction-field charts, but it does not by itself identify their centers after every torsion specialization. For $\widetilde{B}^T \Lambda = (D \mid 0)$ and $N = \text{ord}(q)$, the central classical mutation intrinsically requires $\text{ord}(q^{d_k}) = N$, equivalently $\gcd(N, d_k) = 1$, for every relevant diagonal entry. MNTY's root-of-unity Theorem B uses the half-parameter $\epsilon^{1/2}$ and imposes both coprimality and odd half-parameter order, a sufficient convention-dependent Frobenius hypothesis rather than evidence that parity alone is necessary for Etingof's q . The focal manuscript imposes no full-order condition and calls the chart-center gluing automatic. Whenever some relevant $d_k > 1$, orders divisible by a prime factor of that entry form a Zariski-dense characteristic-zero torsion family, so the missing scope cannot be supplied by the phrase 'generic torsion point'; if all relevant $d_k = 1$, this particular obstruction vanishes. An explicitly restricted full-order family is required in the general case allowed by the theorem.

Verifiable source: Muller-Nguyen-Trampel-Yakimov, exact [Advances in Mathematics](#) version, Theorem B

Losev-Etingof, arXiv:1501.01260v4: The disclosed Theorem 1.1 error is not used by the focal manuscript

Target PDF pages 6-7 · Section 2.6 · arXiv:2504.19329v4

Applicable for the exact cited material

The current arXiv record for the cited paper explicitly says that the proof of its Theorem 1.1 has a crucial error. Section 2.6 of the focal manuscript does not invoke that theorem. It uses the definition of holonomic modules and Lemma 5.1 from the joint appendix concerning isotropic subvarieties under a symplectic resolution; the appendix proves that lemma directly from pullback of the symplectic form. The focal Section 2.6 then states Conjecture 2.14 and a proposed Hamiltonian-reduction route rather than claiming a theorem from the erroneous result. The source defect therefore has no downstream effect here.

Verifiable source: Losev-Etingof, exact [current arXiv](#) version 4

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: [arXiv:2504.19329v4](#)
2. C. Sabbah, Systèmes holonomes d'équations aux q-différences, author PDF
3. Aizenbud-Gourevitch-Minchenko, exact final arXiv version 2
4. T. Bitoun, On the p-supports of a holonomic D-module, *Inventiones Mathematicae* 215 (2019), exact VOR
5. Berenstein-Zelevinsky, Quantum cluster algebras, exact *Advances in Mathematics* version
6. Matherne-Muller, Computing upper cluster algebras, public arXiv version 1 fallback
7. Muller-Nguyen-Trampel-Yakimov, exact *Advances in Mathematics* version, Theorem B
8. Losev-Etingof, exact current arXiv version 4

arXiv acknowledgement

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APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. *Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.*

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use $\$ \dots \$$ for inline mathematics and $\$ \$ \dots \$ \$$ for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.
- Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.
- Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.