MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, novelty, and source audit

Submanifold-genericity of $\mathbb{R}^d$-actions and uniform multiplicative Diophantine approximation

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Prompt	Standardized prompt
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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, novelty, and proof-critical source use

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings
04	Sources	Contains incorrect or incomplete source use

01 / STATEMENTS

Statements

Contains wrong statements

Theorem 1.3 is false for the stated class of semisimple groups and arbitrary lattices: reducible product lattices give invariant factor functions along admissible submanifolds. Requiring the full action to have strong spectral gap, for example by taking an irreducible lattice under the standard hypotheses, repairs it. Theorem 1.4 and the multiplicative conclusions are correct after the verified internal and source repairs recorded below. The displayed measure in Theorem 1.8 needs a local normalization correction.

Theorem 1.3: Reducible product lattices contradict submanifold genericity

PDF page 3 · Theorem 1.3 and estimate (1.4) · arXiv:2504.02258v1

Incorrect as written · verified hypothesis repair

Take $G = G_1 \times G_2$ with $G_i = \mathrm{SL}_3(\mathbb{R})$ and choose cocompact lattices $\Gamma_i < G_i$, setting $\Gamma = \Gamma_1 \times \Gamma_2$. Every simple factor has real rank 2, so the printed hypotheses hold, and $X = X_1 \times X_2$ is compact. Choose nonzero $H \in \mathfrak{a}_1$ and let $M = \{s(H, 0) : 1 \leq s \leq 2\}$, an allowed compact one-dimensional submanifold. For any nonconstant $v \in C^\infty(X_2)$, the function $\varphi(x_1, x_2) = v(x_2)$ belongs to $C_c^\infty(X)$ and every element of tM fixes it. Thus the average in (1.4) equals $v(x_2)$ for every t, while its claimed limit is $\mu_2(v)$; these differ on a positive-measure set. The effective estimate fails as well. The cited BEG theorem explicitly requires strong spectral gap, which this reducible product action lacks. Adding that hypothesis, or imposing the usual irreducibility assumptions that supply it, is the verified repair.

Verifiable source: Björklund–Einsiedler–Gorodnik, Theorem 1.1

Theorem 1.4: Genericity of compact $U_{m,n}$-orbit measure

PDF pages 3 and 9–16 · Theorem 1.4 and Sections 2–3 · arXiv:2504.02258v1

Correct

Here $G = \mathrm{SL}_{m+n}(\mathbb{R})$ is simple and $m + n \geq 3$, so the spectral and irreducibility obstruction affecting Theorem 1.3 is absent. The exact general BG23 theorem applies to the Haar probability on a compact $U_{m,n}$-orbit, whose density has Wiener norm 1. Compactness of $M \subset E$ makes its distance from the cone walls uniformly comparable to the pairwise-separation parameter. The corrected even-moment argument then gives the claimed genericity and, by taking arbitrarily large even moments, the rate $t^{-k/2+\delta}$.

Verifiable source: Björklund–Gorodnik, general Theorem 1.3

Proposition 1.7 and Theorems 1.8–1.9: Uniform multiplicative approximation conclusions survive the proof repairs

PDF pages 6 and 17–30 · Proposition 1.7, Theorems 1.8–1.9, and Sections 4–6 · arXiv:2504.02258v1

Correct

The multiplicative Dani correspondence and the internally proved transference lemma correctly reduce Theorem 1.8 to Theorem 1.4. For Theorem 1.9, the full-measure branch follows after replacing the polytope with a smooth compact patch and supplying the omitted fixed-ratio cusp-annulus estimate; the moment exponents then give the threshold $\lambda < m + n - 2$. In the zero-measure branch, the corrected product-region volume and lattice-point count give $O((\log T)^{m+n-2-\lambda})$. Proposition 1.7 follows from Minkowski after shrinking the two radii within the strict slack $c > m!n!/(m^m n^n)$. These verified repairs preserve the exact parameter ranges of all three conclusions.

Verifiable source: Exact arXiv version 1

Theorem 1.8: The Lebesgue measure must be normalized on a fundamental cube

PDF page 6 · Theorem 1.8 · arXiv:2504.02258v1

Minor formal correction

The set is written as a subset of $\mathbb{R}^{m\times n}$, whose Lebesgue measure is infinite when the set is conull, so the displayed equality $\mathrm{Leb}(D^{\times}_{m,n}(c\psi_1)) = 1$ is not literal. Replace it by

$$\mathrm{Leb}(D^{\times}_{m,n}(c\psi_1) \cap [0,1)^{mn}) = 1,$$

or state that the complement has Lebesgue measure zero. Integer-translation invariance and the proof on the compact $U_{m,n}$-orbit make these formulations equivalent. The correction is local and changes no application.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proof of Theorem 1.3 applies BEG outside its strong-spectral-gap hypothesis and the theorem is genuinely false in that scope. The later proofs have repairable but substantive defects: the chosen averaging patch has corners, the required cusp-annulus measure estimate is absent, two mollifier calculations are false as printed, the zero-measure volume formula is wrong, and the Minkowski argument proves non-strict inequalities. Exact repairs establish the unaffected conclusions. Two literal notation errors are reported separately.

Proof of Theorem 1.3: The cited multiple-mixing theorem lacks a required hypothesis

PDF pages 9–10 · Theorem 2.3 and proof of Theorem 1.3 · arXiv:2504.02258v1

Incorrect as written · no repair at the printed scope

BEG Theorem 1.1 assumes that the acting semisimple group has strong spectral gap on $L^2_0(G/\Gamma)$. The paper replaces that assumption by 'every simple factor has rank at least 2'. Property (T) supplies a gap only when the corresponding factor action is ergodic; for a reducible product lattice, one factor fixes all functions of the other factor. Thus estimate (2.5) is unavailable and the counterexample in the Statements dimension shows that the lost hypothesis cannot be reconstructed. Repair classification: no repair at the printed scope; adding strong spectral gap or an appropriate irreducibility hypothesis repairs the proof and statement.

Verifiable source: BEG exact arXiv version 2, Theorem 1.1

Full-measure proofs of Theorems 1.8–1.9: The averaging polytope is not an allowed C^1 submanifold

PDF pages 22 and 26–28 · definition of M_σ and Lemma 5.4 · arXiv:2504.02258v1

Incorrect as written · verified repair

M_σ is cut out by several simultaneous coordinate inequalities and has corners, whereas Theorems 1.4 and 2.2 assume a C^1 submanifold, possibly with ordinary boundary. The same issue appears in Lemma 5.4, whose parameter set J is a closed polytope although Lemma 3.1 assumes a bounded open chart domain. Replace M_σ by the closure of a small Euclidean ball in the affine slice E_1, compactly contained in E, and take the interior ball as J. The parametrization remains affine, the dimension is $m + n - 2$, and every mixing, collision-volume, and cusp-intersection estimate used later is unchanged up to fixed constants. Repair classification: Verified repair.

Proof of Theorem 1.9: The cusp-annulus measure estimate is missing

PDF page 28 · paragraph beginning 'For Ω_t as in (5.4)' · arXiv:2504.02258v1

Incomplete as written · verified external-input repair

The proof asserts

$$\mu(\Omega_t) \asymp e^{-(m+n)R(t)}$$

and says this follows from Lemma 5.3. That lemma only bounds the support of a mollified indicator; it contains no Haar-measure estimate. Put $D = m + n \geq 3$ and $r = e^{-R(t)}$. The primitive Siegel mean formula together with Rogers' second-moment estimate gives

$$\mu\{\Lambda : \delta(\Lambda) < r\} = \frac{2^{D-1}}{\zeta(D)} r^D + O_D(r^{2D}).$$

Subtracting the same formula at r/e yields $\mu\{r/e \le \delta < r\} \asymp_D r^D$, exactly the missing claim. This repairs the full-measure branch, but it is a substantive proof-critical input and cannot be attributed to Lemma 5.3.

Verifiable source: C. A. Rogers, Mean Values over the Space of Lattices, Theorems 4–5 (printed pages 251–253)

Lemmas 5.1–5.3: The mollifier derivative and support calculations are false as printed

PDF pages 23–26 · equations (5.1)–(5.5) and Lemmas 5.1–5.3 · arXiv:2504.02258v1

Incorrect as written · verified repair

From $\rho_r(\psi(x)) = r^{-D_G}\rho(r^{-1}x)/f(x)$, differentiating along $\gamma_\alpha(x,s) = \psi^{-1}(\exp(s\alpha)\psi(x))$ requires the product and quotient rules. The displayed proof instead differentiates $\rho(r^{-1}\gamma_\alpha/f(\gamma_\alpha))$, moving the density into the argument of ρ and omitting its derivative. Also, coordinate support $x \in B_r(0)$ only gives $\operatorname{supp}\rho_r \subset B_{C_1 r}(\mathrm{id})$, not the opening claim $B_r(\mathrm{id})$, and Lemma 5.3 later reuses the smaller ball. In that lemma the two uniquely determined notation corrections are $\varphi_{r,t} \to \varphi_{t,r}$ and, in the quantified lower bound, $w \to w'$. Applying the correct chain and quotient rules gives $\|D^j\rho_r\|_\infty = O(r^{-D_G-j})$ because f and $1/f$ have bounded derivatives on the chart. Consistently use $B_{C_1 r}$ and initially require $C_1 r < r_0$; all convolution and Sobolev estimates then follow with changed fixed constants. Repair classification: Verified repair.

Zero-measure proof of Theorem 1.9: The displayed product-region volume is not the stated equality

PDF pages 29–30 · final counting argument · arXiv:2504.02258v1

Incorrect as written · verified repair

For $0 < \varepsilon \le 2^{-m}$, the exact volume of

$$\{x \in [-1/2, 1/2]^m : \prod_i |x_i| < \varepsilon\}$$

is

$$2^m\varepsilon \sum_{j=0}^{m-1} \frac{(\log(2^{-m}\varepsilon^{-1}))^j}{j!},$$

not $2^m\varepsilon[(\log(2^{-m}\varepsilon^{-1}))^{m-1} + 1]$ except when $m = 2$. The needed upper bound $O_m(\varepsilon(1 + |\log\varepsilon|)^{m-1})$ nevertheless follows. In the preceding union, replace the unsupported bound $|p| < |q|$ by $|p| \le n|q| + 1$ in the supremum norm after choosing nearest integers; this changes only a fixed counting constant. The corrected count is still $O((\log T)^{m+n-2-\lambda})$. Repair classification: Verified repair.

Proof of Proposition 1.7: Minkowski yields weak inequalities while the target set is strict

PDF page 30 · equations (6.1)–(6.2) and conclusion · arXiv:2504.02258v1

Incorrect as written · verified repair

The closed convex body gives $\prod_i |Y_i q - p_i| \le c/T$ and, when every $q_j \ne 0$, $\Pi_+(q) \le T$, but $S^\times_{m,n}(c\psi_1, T)$ is defined using both strict inequalities. Choose c_0 and η with

$$\frac{m!n!}{m^m n^n} < c_0\eta < c_0 < c,$$

use the first radius with c_0 and replace the second radius by $n(\eta T)^{1/n}$. The body still has volume greater than 2^{m+n}, while the resulting products are $< c/T$ and $< T$ for all sufficiently large T; the cases with zero coordinates are already easier. Repair classification: Verified repair.

Proof of Theorem 2.2: The mean-value point has the wrong sign

PDF page 11 · proof of Theorem 2.2 · arXiv:2504.02258v1

Typo

The printed point $\theta_j s-(1-\theta_j)s'$ should be $\theta_j s+(1-\theta_j)s'$. The latter lies on the segment joining s' to s, as required by the mean value theorem. Every downstream estimate uses only membership in that convex segment, so the correction is unique and harmless.

Proof of Theorem 1.9: The subscripts of the approximation function are reversed

PDF pages 29–30 · three occurrences in the zero-measure argument · arXiv:2504.02258v1

Typo

Replace each $\psi_{\lambda,1}$ by $\psi_{1,\lambda}$. Only $\psi_{1,\lambda}(x)=x^{-1}(\log x)^{-\lambda}$ is defined, and that substitution is exactly what the displayed estimate uses.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

04 / SOURCES

Sources

Contains incorrect or incomplete source use

The BEG theorem is invoked without its strong-spectral-gap hypothesis, producing a false headline theorem. The BG23, BG20, and FK inputs are mathematically sufficient after correcting the BG23 theorem number and the FK proposition/corollary label, while the fixed-ratio cusp-annulus estimate is a necessary uncited input. Thus the focal source chain is neither correct nor complete as printed.

BEG Theorem 1.1: Strong spectral gap is indispensable, not implied by the printed rank condition

Target PDF pages 3 and 9–10 · Theorems 1.3 and 2.3 · arXiv:2504.02258v1

Incorrect source use

The exact BEG theorem assumes that the G-action on G/Γ has strong spectral gap, meaning that its restriction to every noncompact simple factor is isolated from the trivial representation on L^2_0. The target drops this and assumes only rank at least 2 for each factor. For a product lattice, each factor fixes the nonconstant functions pulled back from the other quotient, so the source hypothesis fails. The explicit counterexample in the Statements dimension proves that the source conclusion cannot be extended as claimed. Adding strong spectral gap or an appropriate irreducibility condition is necessary.

Verifiable source: Quantitative Multiple Mixing, arXiv:1701.00945v2, Theorem 1.1

BG23 general Theorem 1.3: The cited theorem number is too narrow, but the same paper has the needed general result

Target PDF page 10 · Theorem 2.4 · arXiv:2504.02258v1; BG23 VOR pages 213–214

Applicable after verified citation repair

The target attributes its arbitrary-lattice statement to BG23 Theorem 1.1. In the exact version of record, Theorem 1.1 is the special $\mathrm{SL}_{m+n}(\mathbb{R})/\mathrm{SL}_{m+n}(\mathbb{Z})$ case. The required parabolic-unipotent statement is general Theorem 1.3. It applies here because $G = \mathrm{SL}_{m+n}(\mathbb{R})$ is simple, every lattice is irreducible in the relevant sense, $U_{m,n}$ is the abelian unipotent radical, and Haar probability on its compact orbit is a Wiener measure of norm 1. Replacing the locator 'Theorem 1.1' by 'Theorem 1.3' verifies the source use.

Verifiable source: Effective Multiple Equidistribution of Translated Measures, version of record

Fixed-ratio cusp-annulus asymptotic: A proof-critical geometry-of-numbers input is absent

Target PDF page 28 · proof of Theorem 1.9 · arXiv:2504.02258v1

Missing proof-critical source

Neither Lemma 5.3 nor any represented citation proves the asserted estimate $\mu(\Omega_t) \asymp e^{-(m+n)R(t)}$. This is not a support property of the mollifier; it is a Haar-measure theorem for short primitive lattice vectors. Rogers' primitive first- and second-moment formulas, Theorems 4–5, supply the exact asymptotic and repair the argument, as detailed in the Proofs dimension. Because this external result is necessary for the lower bound in Lemma 5.4 and hence the full-measure half of Theorem 1.9, it must be represented as a direct dependency rather than treated as routine algebra.

Verifiable source: C. A. Rogers, Mean Values over the Space of Lattices, Theorems 4–5 (printed pages 251–253)

BG20 Proposition 6.2 and FK24 Lemma 4.1 / Proposition 4.4 / Corollary 4.5: The covering and multiplicative-correspondence inputs are applicable after one locator correction

Target PDF pages 12 and 17–18 · Lemma 3.4, Lemma 4.1, and Proposition 4.2 · arXiv:2504.02258v1

Applicable and sufficient after verified citation repair

BG20 Proposition 6.2 is the exact finite cluster decomposition used in Lemma 3.4; its proof uses only the metric triangle inequality, so the target's extension from the source's group notation to a metric space is valid. The exact source available when the target appeared is arXiv:2211.04523v3: its Lemma 4.1 gives the ψ–R parametrization, Proposition 4.4 gives precisely the multiplicative Dani correspondence used in Proposition 4.2 and Corollary 4.3, and Corollary 4.5 is the cited consequence. The target's locator 'Proposition 4.5' must therefore be corrected to 'Corollary 4.5'. Its sign convention for the last n diagonal exponents is a faithful renaming. Version 4 appeared later on April 3, 2025 and is outside the represented source boundary; the unrelated defects found elsewhere in version 3 do not affect Lemma 4.1, Proposition 4.4, or Corollary 4.5 as used here.

Verifiable source: Fregoli–Kleinbock, exact cited arXiv version 3

Minkowski convex body theorem: The theorem applies; strictness is an internal slack issue

Target PDF page 30 · proof of Proposition 1.7 · arXiv:2504.02258v1

Applicable and sufficient after the verified internal repair

The body $\Xi(Y, T, c)$ is convex, centrally symmetric, and has the displayed volume $2^{m+n}m^m n^n c/(m!n!)$. Minkowski therefore supplies a nonzero integer point once $c > m!n!/(m^m n^n)$. The source theorem itself is sufficient. The mismatch between weak inequalities from the closed body and strict inequalities in $S^{\times}_{m,n}$ is repaired entirely inside the focal proof by the two slack parameters described in the Proofs dimension.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty, and the correctness and completeness of proof-critical source use. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2504.02258v1
2. Quantitative Multiple Mixing, arXiv:1701.00945v2, Theorem 1.1
3. Effective Multiple Equidistribution of Translated Measures, version of record
4. Exact arXiv version 1
5. C. A. Rogers, Mean Values over the Space of Lattices, Theorems 4–5 (printed pages 251–253)
6. Fregoli–Kleinbock, exact cited arXiv version 3

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt and in-depth source supplement

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

IN-DEPTH SOURCE EVALUATION — FOCAL PAPERS ONLY

Apply this supplement only when the audit scope is explicitly marked focal-in-depth. Do not apply it to ordinary audits of prerequisite papers.

Evaluate whether every external result materially used by the focal paper is both correctly invoked and sufficient for the exact downstream claim. Trace each such use to the exact cited source version represented in the dependence graph. Check the cited statement's hypotheses, conclusion, quantifiers, parameter ranges, and any translation of notation. Also check completeness: identify a missing proof-critical source only when a concrete step in the focal paper requires an external result that is neither proved there nor supplied by the represented sources.

Report this assessment as a fourth dimension titled Sources, after Statements, Proofs, and Novelty. Use exactly one overall status:

- Sources correct and complete: every proof-critical external input checked is applicable and sufficient, and no missing proof-critical source was identified.

- Contains unverified source use: no incorrect or incomplete source use was established, but at least one material source obligation could not be verified from the exact available versions.

- Contains incorrect or incomplete source use: at least one cited result is inapplicable or insufficient as used, or a concrete proof-critical source is missing.

Do not report citations used only for background, comparison, attribution, examples, or literature review. Do not turn this category into a bibliography audit. Apply the same evidentiary threshold, immediate-consequence closure, location precision, repair discipline, and LaTeX rules as the standardized audit prompt.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.