MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Metric theory of inhomogeneous Diophantine approximations with a fixed matrix

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Prompt	Standardized prompt
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Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The central fixed-matrix transference, measure, winning, and Khintchine-type results are correct. Four material arguments are not valid or complete as printed: the lacunarity pigeonhole argument, the metric Cassels overlap count, the local independence normalization, and the claim that different subsequences automatically yield different infinite sums. Explicit repairs below verify the stated conclusions.

Theorems 3.1–3.4: Uniform and asymptotic fixed-matrix transference results

Pages 12–15 and 28–36 · Theorems 3.1–3.4 and their general forms · arXiv:2503.21180v3

Correct

The transference inequalities correctly convert homogeneous approximants for $\Theta^\top$ into restrictions on the scalar products of a shift with the dual vectors. The Borel–Cantelli argument on non-exceptional affine subspaces proves the null conclusions, and the lacunary hyperplane strategy establishes the HAW conclusions. The local independence ratio in Lemma 6.5 must be normalized by the reciprocal ball measure, as the downstream Chung–Erdős computation already requires; the slicing repair below supplies that normalization and verifies the measure argument.

Theorems 3.6 and 3.7: Metric Kurzweil-type null statements on affine slices

Page 16 and pages 34–39 · Theorems 3.6–3.7 and proofs · arXiv:2503.21180v3

Correct

The summability hypothesis removes the scalar-product exceptional set, while the transference inequality turns any too-good inhomogeneous approximation into one of those excluded events. The metric Cassels lemma supplies the complementary full-measure limsup statement. After replacing its one-dimensional overlap count by the coordinatewise n-dimensional count below, the second-moment estimate is valid and the advertised affine-slice conclusions follow with the stated constants.

Theorem 3.8 and Theorem 9.1: Khintchine-type comparison and uncountability

Page 17 and pages 39–42 · Theorem 3.8, Theorem 9.1, and proof · arXiv:2503.21180v3

Correct

Part (a) follows by subtracting consecutive inhomogeneous best approximants. For part (b), Proposition 5.1 supplies a sufficiently sparse subsequence whose denominator sums and error tails are geometrically dominated, giving the factor $2B$. To justify uncountability, choose the candidate errors in consecutive pairs with each candidate at most one sixteenth the norm of its predecessor, and choose one member of every pair. The first pair at which two binary choices differ contributes more than the combined later tails, so the sums are distinct. This repairs the printed implication and proves that there are uncountably many shifts satisfying the estimate.

Proposition 5.1: Uniform growth and decay of best approximations

Pages 23–24 · Proposition 5.1 and proof · arXiv:2503.21180v3

Correct

Part (a) is the cited standard growth bound. For part (b), pair the $2^{2n} - 2^n$ small remainder cubes into antipodal pairs. Among $B + 1$, with $B = 2^{m-1}(2^{2n} - 2^n)$, one antipodal pair contains at least $2^m + 1$ remainders. Change signs so all of them lie in the same member of the pair, and partition the signed denominator box into 2^m half-size boxes. Two denominators then share a box; their nonzero difference has norm at most $P_{\nu+B}$ and error at most $\psi_\Theta(P_\nu)/2$. Hence

$$\psi_\Theta(P_{\nu+B}) \leq \frac{1}{2}\psi_\Theta(P_\nu),$$

as stated.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

Several proofs require material repair. Proposition 5.1 mixes numerator remainders with full lattice vectors; Lemma 7.9 uses a one-dimensional ceiling in an n-dimensional count and then inserts extraneous factors; Lemma 6.5 states the wrong normalization without proving the pair estimate; and Theorem 9.1's uncountability inference is false for arbitrary nonzero summands. Each gap has a verified repair that preserves the theorem statements.

Proposition 5.1(b): The pigeonhole argument uses incompatible spaces and signs

Pages 23–24 · proof of Proposition 5.1(b) · arXiv:2503.21180v3

Incorrect as written · verified repair

The remainders $\Theta\mathbf{p}_j - \mathbf{a}_j$ are n-vectors, but the proof places them in boxes in $\mathbb{R}^{m+n}$. It then adds both signs of selected denominators without ensuring that the signed remainders remain in one small cube. Instead, partition the n-dimensional remainder shell into $2^{2n} - 2^n$ cubes and pair antipodal cubes. One pair contains at least $2^m + 1$ remainders; orient each by a sign into the same cube. Two of the corresponding signed denominators occupy the same one of 2^m half-size denominator boxes. Their difference is nonzero because distinct best-approximation indices have distinct norms, and it has exactly the bounds stated in the statements audit. The final printed inequality also has an extra factor $1/2$ on its left; deleting it yields the proposition.

Lemma 6.5: The independence ratio is inverted and its derivation is omitted

Page 27 · Lemma 6.5 · arXiv:2503.21180v3

Incomplete as written · verified repair

If B is the fixed ball, asymptotic independence relative to normalized measure means

$$\frac{m(U_k \cap U_s \cap B)}{m(U_k \cap B)m(U_s \cap B)} \longrightarrow \frac{1}{m(B)},$$

not $m(B)$. The reciprocal is also the value used later to make the Chung–Erdős lower bound equal to $m(B)$. To prove it, decompose B into slabs for the lower-frequency functional, apply the preceding uniform strip estimate to the higher-frequency functional on every complete slab, and bound the two boundary fragments. The condition $|\operatorname{pr}_{\mathcal{L}} u_s|/|\operatorname{pr}_{\mathcal{L}} u_k| \to 0$ makes the total boundary contribution negligible uniformly in the lower-frequency slab. This gives the displayed reciprocal limit and completes the omitted step.

Lemma 7.9: The metric Cassels overlap count must be coordinatewise

Pages 36–37 · proof of Lemma 7.9, especially display (7.27) · arXiv:2503.21180v3

Incorrect as written · verified repair

The number of n-dimensional grid boxes meeting a fixed cube is estimated by a single ceiling, and the next double sum contains extra factors $W_s'W_k'$. Coordinatewise counting instead gives, for sufficiently sparse k, at most

$$(1+\varepsilon)W_k' m(I_s)$$

fine cubes meeting one fixed coarse cube. Since the fine cubes are disjoint, summing their overlap volumes over the W_s' coarse cubes yields

$$m(E_s \cap E_k) \le (1+\varepsilon)W_s'W_k' m(I_s)m(I_k) = (1+\varepsilon)m(E_s)m(E_k).$$

The Chung–Erdős lemma then gives the claimed full-measure limsup.

Theorem 9.1(b): Different subsequences need not produce different sums

Page 41 · last paragraph of the proof of Theorem 9.1 · arXiv:2503.21180v3

Incomplete as written · verified repair

Nonzero summands alone do not make the map from subsequences to infinite sums injective. Thin the liminf-realizing candidate sequence so every error vector has norm at most one sixteenth of its predecessor and all index-gap requirements remain satisfied. Group candidates in pairs and select one candidate from each pair. At the first differing pair, the norm of the difference of the two selected terms is at least fifteen sixteenths of the larger term, whereas the sum of both later tails is less than two fifteenths of it. The resulting sums are therefore distinct, and the binary choices give continuum many valid shifts.

Exceptional-set and complement notation: Two set-theoretic displays have unique corrections

Pages 25 and 13 · exceptional-set definition and discussion after Theorem 3.2 · arXiv:2503.21180v3

Typo

The exceptional set defined by infinitely many occurrences requires the limsup $\bigcap_N \bigcup_{\nu \geq N}$, not a single union. Also, when bad approximability forces only the trivial singular shifts, it is the singular set itself that is countable, not its complement; the complement was just proved HAW. The definitions and adjacent conclusions determine both corrections uniquely.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2503.21180v3

arXiv acknowledgement

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

$$...$$

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.