MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

The Dirichlet spectrum

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Prompt	Standardized prompt
Novelty cutoff	December 8, 2024
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The interval theorem for Dirichlet spectra in every matrix dimension other than $(1,1)$, its weighted lattice-flow extension, and the stated ψ-spectrum consequences are correct. The printed upper-uniformity construction fails at rational line parameters, but a direct uniform Dirichlet estimate repairs both affected applications. The remaining notation errors are mechanically identifiable as typos.

Theorem 1: The full Dirichlet spectrum is an interval outside dimension $(1,1)$

Page 2 and pages 6–14 · Theorem 1 and its deduction from Theorems 2 and 5 · arXiv:2412.05858v1

Correct

For the compacta $K_\varepsilon = \{\Lambda : \lambda_1(\Lambda) \geq \varepsilon\}$, the extended threshold equals the limsup of the first minimum along the diagonal orbit. Lemma 2 identifies its appropriate power with the limsup defining the Dirichlet constant. The topological theorem produces every interior value from the dense full-measure set of maximal limsup points and the dense connected family on which the limsup is zero. The zero endpoint is realized on those rational affine families, while the maximal endpoint is realized on a full-measure set. This gives $\mathbb{D}_{m,n} = [0, \Delta]$ and uncountably many realizations in every open set when $\max(m,n) > 1$.

Theorem 2: Weighted extended spectra for arbitrary compact exhaustions

Page 3 and pages 10–14 · Theorem 2 and Section 5.1 · arXiv:2412.05858v1

Correct

Composing the continuous exhaustion gauge with $g_{t,\boldsymbol{\alpha},\boldsymbol{\beta}}\Lambda_\Theta$ gives the continuous function required by Theorem 5. For $n = 1$, rational affine lines supply the upper-uniform family. The direct Dirichlet estimate recorded under Proofs is uniform in the line parameter and tends to zero because $\alpha_{\max} < 1$. For $n \geq 2$, rational affine planes $\Theta\mathbf{i} = \mathbf{z}$ contain a fixed contracting lattice vector. Kleinbock–Weiss equidistribution shows that almost every expanding-horosphere point visits every E_c with $c < b$ at arbitrarily large times. The endpoint arguments then give the entire interval $[0, b]$.

Theorem 3 and Corollary 1: The ψ-Dirichlet spectrum and prescribed uniform exponent

Pages 4–5 and 14–20 · Theorem 3, Corollary 1, and technical lemmas · arXiv:2412.05858v1

Correct

The same connected-family argument applies to $\lambda_{\Theta,\psi}$. When $n = 1$, the additional condition $t^m\psi(t) \to \infty$ is exactly $\psi(t)^{-1/m} = o(t)$; the repaired direct Dirichlet estimate under Proofs then tends uniformly to zero. For $n \geq 2$, the fixed rational kernel vector contracts as $t^{-1/n}$. Lemma 1 transfers the almost-everywhere positive limsup for the Dirichlet scale t^{-1} to infinite limsup at the smaller function $\psi = o(t^{-1})$, supplying the dense maximal set. Lemma 2 then converts the power-law choice of ψ into the claimed simultaneous prescription of the uniform exponent and the critical limsup.

Definitions of χ and χ_γ: The approximating vector is printed in the wrong ambient set

Pages 1 and 4 · definitions of $\chi(\Theta, t)$ and $\chi_\gamma(\Theta, t)$ · arXiv:2412.05858v1

Typo

Both displays place $\mathbf{p}$ in $\mathbb{R}^m$. Literally, the minimum would then be zero by choosing $\mathbf{p} = \Theta\mathbf{q}$. Every surrounding statement, formula (4), the definition of $\lambda_{\Theta,\psi}$, Lemma 2, and every proof instead use $\mathbf{p} \in \mathbb{Z}^m$. Replacing $\mathbb{R}^m$ by $\mathbb{Z}^m$ in the two displays is therefore a uniquely determined symbol correction. Under the audit's typo rule, it does not make the central theorems incorrect.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The topological interpolation theorem and the technical comparison lemmas are correct. In both $n = 1$ homogeneous-space applications, however, the printed proof asks for an infinite strictly improving approximation sequence even when the line parameter is rational, where no such sequence exists. A direct Dirichlet estimate gives a verified uniform repair. The remaining notation and indexing slips are local and harmless.

Theorem 5: Nested connected interpolation realizes every intermediate limsup

Pages 8–10 · Theorem 5 and its proof · arXiv:2412.05858v1

Correct and complete

At each stage a point with value above c is joined, inside one connected member of the distinguished family, to a point whose entire late-time tail is below $c - \varepsilon_k$. The two closed subsets defined by staying below $c - \varepsilon_{k+1}$ and reaching at least c cannot separate that connected set, so an intermediate point and a smaller neighborhood satisfy both the upper bound and a near-c witness. Compactly nested closures give a common point; the covering time intervals bound its eventual values by c, and the witnesses force limsup c. Diagonal avoidance of a purported enumeration proves uncountability.

The $n = 1$ upper-uniformity estimates: The strictly improving sequence does not exist for rational parameters, but a direct estimate repairs both uses

Pages 11–15 · $n = 1$ cases in the proofs of Theorems 2 and 3 · arXiv:2412.05858v1

Incorrect as written · verified repair

Both applications choose an infinite sequence q_k with strictly decreasing errors $|q_k y - p_k|$. When $y \in \mathbb{Q}$, an exact zero error occurs and no infinite strictly decreasing sequence exists, so the subsequent balancing times are undefined; rational values of y do occur on the chosen affine lines. For any integer $N \geq 1$, Dirichlet's theorem instead gives $1 \leq q \leq N$ and $|qy - p| \leq 1/N$, uniformly in y. In the proof of Theorem 2, take N comparable to $t^{(1+\alpha_{\max})/2}$. Then the constructed lattice vector satisfies

$$\max\left\{\frac{q}{t},\ i_{\max} t^{\alpha_{\max}} |qy - p|\right\} \leq C t^{(\alpha_{\max}-1)/2} \longrightarrow 0.$$

In the proof of Theorem 3, take N comparable to $\left(t\psi(t)^{-1/m}\right)^{1/2}$ to obtain

$$\max\left\{\frac{q}{t},\ i_{\max} \psi(t)^{-1/m} |qy - p|\right\} \leq C \left(\frac{\psi(t)^{-1/m}}{t}\right)^{1/2} \longrightarrow 0.$$

The constants are fixed on each rational affine line, so both estimates are uniform on compact subsets and verify the required upper uniformity for rational and irrational y at once. The two auxiliary claims should correspondingly require their threshold to be uniform on compact subsets; the repaired estimates satisfy that corrected premise. For $n \geq 2$, the printed fixed-kernel-vector argument is unaffected, and the equidistribution step supplies the dense maximal-limsup set.

Lemmas 1–4: Successive minimizers and limsup comparison

Pages 16–20 · technical lemmas and proofs · arXiv:2412.05858v1

Correct and complete

Successive minimizing pairs have strictly increasing denominator norms and strictly decreasing errors. Their balancing times are precisely the local maxima of $\lambda_{\Theta,\psi}$. If $\lambda_{\Theta,\psi}$ stayed bounded while $\psi = o(t^{-1})$, the products $\|\mathbf{q}_{k+1}\|^n \|\Theta \mathbf{q}_k - \mathbf{p}_k\|^m$ would tend to zero; balancing the same pairs at the Dirichlet scale would then force $\limsup \lambda_{\Theta,t^{-1}} = 0$, a contradiction. The analogous balancing calculation for ψ_γ yields exactly the exponent $1 + \gamma n/m$ and proves the comparison with $\chi_{\gamma n/m}$.

Section 5.2 line family: One occurrence of $\mathbb{Z}^m$ must be $\mathbb{Q}^m$

Page 14 · first paragraph of the $n = 1$ case in the proof of Theorem 3 · arXiv:2412.05858v1

Typo

The proof says "as before" but prints the line family with both $\mathbf{i}, \mathbf{z} \in \mathbb{Z}^m$; the earlier locally connected family has $\mathbf{i} \in \mathbb{Z}^m$ and $\mathbf{z} \in \mathbb{Q}^m$. The subsequent sentence invokes the earlier local-connectedness proof and the construction again chooses a denominator clearing $\mathbf{z}$, so $\mathbb{Q}^m$ is the intended set. Restoring it makes the cited argument apply verbatim.

Proof of Theorem 5: Two compact-set and enumeration indices are locally misstated

Pages 8–10 · base step and final uncountability paragraph · arXiv:2412.05858v1

Minor formal correction

Upper uniformity applies on compact sets, so the base-step bound should be stated on $X_{j_0} \cap \overline{\Omega_0}$ rather than all of X_{j_0}; this is exactly the set used later. In the diagonal argument, exclude v_k when choosing the kth smaller neighborhood rather than the printed v_{k+1}. Both corrections are internal to the proof, mechanically preserve its induction, and change no theorem statement.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2412.05858v1

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ *is proved for every sufficiently small* $delta > 0$ *and* P *is immediately monotone in the required direction, then the claim that* $P(delta)$ *holds for every* $delta > 0$ *requires no correction; the omitted monotonicity step is exposition rather than an audit finding.*

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ *and subsequently applied at* $t = 0$ *remains a formal scope mismatch unless the endpoint case was separately established before the application.*

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

$$...$$

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.