

Correctness, proof completeness, and novelty audit

Real convergence and periodicity of p -adic continued fractions

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Prompt	Standardized prompt
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Paper license	CC BY-SA 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

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01 / STATEMENTS

Statements

Contains wrong statements

Proposition 1, the necessary real-convergence condition for periodic Browkin-type expansions, is correct. The probabilistic part is not supported by a defined probability law on quadratic irrationals, and Theorem 26's conditional and unconditional expectation formulas are false even under the independent uniform-digit model used implicitly in its proof.

Proposition 1: Eventual periodicity forces convergence to a real conjugate

Pages 3 and 12-14 · Proposition 1 and its proof · arXiv:2410.09215v3

Correct

For a purely periodic tail of period k , the two real conjugates are the fixed points of the period matrix. Lane's criterion reduces possible Euclidean nonconvergence to equality of their distances from A_{k-1}/B_{k-1} , since a finite convergent is rational and cannot equal either quadratic fixed point. The equality is equivalent to $A_{k-1} + B_{k-2} = 0$. Substitution into the discriminant and the determinant identity gives $D = 4(-1)^{k-1} > 0$, so it can occur only for odd k . Browkin II has even period because its integer and noninteger partial quotients alternate. For Browkin I and Algorithm (12), the valuation identities imply that the equality would require $v_p(a_0) + v_p(a_{k-1}) = 0$, while the algorithmic valuation conditions make this sum negative in the only remaining odd-period cases. Reattaching a finite preperiod is a rational linear-fractional substitution and sends the two real tail values to the two real conjugates of the original quadratic irrational.

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Assumption 3 and Propositions 22-24: The claimed probability law on quadratic irrationals is not defined

Page 4 · Assumption 3; pages 17-19 · Remark 21 and Propositions 22-24 · arXiv:2410.09215v3

Not able to verify

The set of p -adic numbers quadratic over $\mathbb{Q}$ is countable, because it is contained in the union of the root sets of the countably many quadratic polynomials in $\mathbb{Q}[x]$. It consequently has Haar measure zero in $\mathbb{Q}_p$, so restricting Haar measure to quadratic irrationals does not yield the probability measure invoked in Assumption 3. Moreover, the displayed assumption specifies only the one-coordinate marginals $\mathbb{P}(c_n = j) = 1/p$; it specifies neither joint independence of digit blocks nor an invariant law for the complete quotients produced by Browkin I. Thus the random variables in Propositions 22-24 have no stated probability law in the advertised quadratic-irrational setting, and their distributional claims cannot be verified from Assumption 3.

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Proposition 24: The expected valuation has the wrong sign

Pages 18-19 · Proposition 24 and its first displayed line of proof · arXiv:2410.09215v3

Typo

Proposition 22 assigns the value $v_p(\alpha_n) = -k$ with probability $(p-1)/p^k$. Therefore its own distribution gives

$$\mathbb{E}(v_p(\alpha_n)) = -\sum_{k \geq 1} k \frac{p-1}{p^k} = -\frac{p}{p-1},$$

not $p/(p-1)$. Insert the missing minus sign in the statement and throughout the calculation. The result is not used in the proof of Theorem 26, and Remark 25 already describes the valuation as negative, so this correction is mechanically determined and has no downstream effect.

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Theorem 26: Both expectation formulas use the wrong conditional sample space

Pages 19-21 · Theorem 26 and proof · arXiv:2410.09215v3

Incorrect

Even under the independent balanced-digit model implicitly used by the proof, conditioning on $v_p(a) = -k$ requires the leading digit c_k in $a = c_0 + c_1/p + \dots + c_k/p^k$ to be nonzero. The proof instead averages over all p^{k+1} digit strings, including $c_k = 0$. For the decisive case $p = 3$ and $k = 1$, the six conditional values are $-4/3, -2/3, -1/3, 1/3, 2/3, 4/3$, so

$$\mathbb{E}(|a| \mid v_p(a) = -1) = \frac{7}{9},$$

whereas the theorem gives $20/27$. Direct summation for general p and k gives

$$\mathbb{E}(|a| \mid v_p(a) = -k) = \frac{p^{2k+1} + 1}{4p^{2k}},$$

and, with the distribution asserted in Proposition 22,

$$\mathbb{E}(|a|) = \frac{p}{4} + \frac{1}{4(p^2 + p + 1)}.$$

These differ from both printed formulas, so Theorem 26 is false under the natural strengthened model as well as unsupported under Assumption 3.

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02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proof of Proposition 1 is correct after immediate-consequence closure. The probabilistic proof chain is not valid: Proposition 22 uses independence absent from Assumption 3, and Theorem 26 averages over a sample space inconsistent with its conditioning event. Two local sign or index defects are separately identified as typographical.

Proof of Proposition 1: The period-matrix and valuation argument closes

Pages 12-14 · proof of Proposition 1, equations (20)-(23) · arXiv:2410.09215v3

Correct and complete

The proof excludes the equimodular fixed-point case through the determinant identity and then uses exactly the parity and valuation restrictions supplied by each of the three algorithms. The additional clause in Lane's criterion is automatic because every finite convergent is rational while the fixed points are quadratic irrationals. The finite preperiod is handled by the same rational linear-fractional relation used to define complete quotients, an immediate formal consequence that preserves the two conjugate values.

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Proof of Proposition 22: Marginal digit uniformity is used as block independence

Page 18 · proof of Proposition 22, immediately before equation (25) · arXiv:2410.09215v3

Incomplete as written

The step

$$\mathbb{P}(c_1 = \dots = c_{k-1} = 0) = p^{-(k-1)}$$

does not follow from the only stated hypothesis $\mathbb{P}(c_i = j) = 1/p$. For example, if a single uniform variable U in $\{0, \dots, p-1\}$ is used for every digit, then every marginal is uniform but the displayed block probability is $1/p$, not $p^{-(k-1)}$. The proof also multiplies that block probability by $\mathbb{P}(c_k \neq 0)$ without establishing independence, and it does not show that the Browkin complete-quotient transformation preserves any proposed law. Repair classification: No repair supplied. A repair would require a precisely defined probability space, the necessary joint digit law at every complete quotient, and a justification connecting that law to quadratic irrationals; none follows from Assumption 3.

Verifiable source: [Full paper, version 3](#)

Proof of Theorem 26: The recurrence computes an unconditional digit average

Pages 20-21 · proof of Theorem 26, recurrence for E_k and the final averaging step · arXiv:2410.09215v3

Incorrect as written

The recurrence defines E_k by averaging $|c_0 + c_1/p + \dots + c_k/p^k|$ over every choice of $c_0, \dots, c_k$. The conditional expectation stated in the theorem is instead over the strict subset $c_k \neq 0$, because that condition is exactly $v_p(a) = -k$. The base case already exposes the mismatch: for $p = 3$ and $k = 1$, excluding $c_1 = 0$ gives $7/9$, while the recurrence including it gives $20/27$. Every subsequent substitution into the sum over k therefore propagates the wrong quantity. Repair classification: Verified repair under an expressly independent uniform-digit model is

$$\mathbb{E}(|a| \mid v_p(a) = -k) = \frac{p^{2k+1} + 1}{4p^{2k}}$$

and

$$\mathbb{E}(|a|) = \frac{p}{4} + \frac{1}{4(p^2 + p + 1)}.$$

This calculation does not repair the separate absence of such a model for quadratic irrationals.

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Proof of Proposition 1: One denominator in the reformulation of equation (20) has the wrong index

Page 13 · display immediately before equation (22) · arXiv:2410.09215v3

Typo

The displayed fraction $(A_{k-1} + B_{k-2})/B_{k+1}$ must have denominator B_{k-1} . This is forced by the two fractions in the preceding line. Only the vanishing of the numerator is used afterward, so the uniquely determined correction does not affect the proof.

Verifiable source: [Full paper, version 3](#)

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. [Full paper, version 3](#)

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. *Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.*
2. *Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.*

3. *Immediate-consequence closure*: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as *Typo*, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as *Minor formal correction*, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be *Correct in Part 1* because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an *Incorrect theorem classification*.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.