MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

On the Factor Complexity Associated with a Family of Multidimensional Continued Fraction Algorithms

Thomas Garrity, Otto Vaughn Osterman

Paper	arXiv:2410.02032v2
Source version	Submitted 21 July, 2026; v1 submitted 2 October, 2024; announced October 2024.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	October 2, 2024 (all audited central results already appear in arXiv v1)
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The triangle-map bound, its conjugacy and twinning transfers, the degenerate Sturmian cases, the finite-witness exclusions, and the complete complexity trichotomy for the $T(e, 13, e)$ class are correct. The proof defects identified below all have verified repairs and do not make a central statement false or unsupported.

Theorem 2: Triangle-map languages have complexity between $2n + 1$ and $3n$

Pages 12–27 · Theorem 2 and Section 6 · arXiv:2410.02032v2

Correct

Rational independence gives the lower bound through Tijdeman's general frequency-complexity result. For the upper bound, the antecedent construction exhausts the non-neutral bispecial factors, their multiplicities occur in ordered $+1, -1$ pairs, and the second-difference identity gives $p_{\mathcal{L}}(n + 1) - p_{\mathcal{L}}(n) \leq 3$. The false three-matrix positivity assertion listed in Part 2 has a complete repair; after that repair the argument proves the full stated range $n \geq 1$.

Verifiable source: Full paper, version 2

Theorems 3–6 and 8: Equivalence transfers, low-complexity classes, and finite-witness exclusions

Pages 28–31 · Theorems 3–6 and 8 · arXiv:2410.02032v2

Correct

Relabeling letters and reversing words preserve factor complexity, so the $T(e, e, e)$ and Cassaigne results transfer to their listed classes. In each degenerate class the isolated letter is fixed and the restriction to the other two letters is a standard Sturmian directive system; rational independence excludes an eventually one-sided directive. The 14 witnesses in Theorem 6 and all 18 witnesses in Theorem 8 were independently recomputed from the displayed substitutions: every listed word has the printed number of distinct length-n factors, strictly exceeding $3n$. The extra $T(e, e, 123)$ exception in Theorem 6 is therefore harmless to the weaker theorem as printed and is repaired in Part 2.

Verifiable source: Cassaigne–Labbé–Leroy input

Theorems 7 and 9: Complete complexity trichotomy for $T(e, 13, e)$

Pages 30 and 35–38 · Theorems 7 and 9 and Lemmas 5–7 · arXiv:2410.02032v2

Correct

Every nonempty right-special factor is of exactly one of two terminal types. De-substitution makes the factors of each type precisely the finite suffixes of one nested limit word. Conditions (I') and (II') are equivalent to finiteness of the corresponding limits, while failure of a condition makes the relevant lengths unbounded. Hence there are two, one, or zero right-special factors at each length, and summing the first differences gives exactly $2n + 1$, $\min\{2n + 1, n + c\}$, or $\min\{2n + 1, n + c_1, c_2\}$. The defective induction sentence in Proposition 22 has the verified replacement stated in Part 2.

Verifiable source: Full paper, version 2

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

Two printed proof steps are formally incorrect and one classification list contradicts its own verified witness table. Each has a complete local repair, so the theorem statements remain verified.

Proposition 3: Three consecutive Gauss incidence matrices need not have all positive entries

Pages 12–13 · Proposition 3 and its displayed three-matrix product · arXiv:2410.02032v2

Incorrect as written · verified repair

The displayed product has $(2, 1)$-entry k_1, so it is zero when $k_1 = 0$; in particular, the assertion that every such three-matrix product is strictly positive is false. The required primitivity nevertheless follows from four consecutive matrices. In the only problematic case, the second row after three factors is $[0, 1, 1]$, and multiplying by M_{k_3} gives $[1, 1, 1]$; all other rows were already positive and remain positive. Thus every four-factor block is strictly positive, which proves Proposition 3 and repairs every downstream use.

Verifiable source: Full paper, version 2

Proposition 22(b): The printed induction for excluding 22 and 33 uses an invalid equivalence

Page 36 · Proposition 22(b), item (iii), and the paragraph following it · arXiv:2410.02032v2

Incorrect as written · verified repair

Item (iii) compares occurrence of 33 and 22 in the same iterated image, while the following explanation treats an occurrence of 33 as though it always came from 12 or 22 in the de-substituted word. That boundary description also depends on whether the outer parameter is zero. A simultaneous induction repairs the proof: 22 in an outer image can only come from 33 in the inner word; 33 can occur only when the outer parameter is 0 and then requires 12 or 22 in the inner word; and 12 is already forbidden. Starting with the one-letter images, neither 22 nor 33 can therefore occur at any depth. This proves Proposition 22(b) and restores the input used in Theorem 9.

Verifiable source: Full paper, version 2

Theorem 6 exception list: $T(e, e, 123)$ is excluded in the statement but certified by the proof table

Pages 29–30 · Theorem 6, witness table, and final sentence of its proof · arXiv:2410.02032v2

Incorrect as written · verified repair

The theorem lists $T(e, e, 123)$ among the exceptions, but its own table gives the directive prefix 11110010 with $n = 2$ and seven distinct factors. Independent substitution and factor enumeration reproduces $p_w(2) = 7 > 6$. Consequently the sentence saying that the theorem's exceptions are precisely the cases absent from the table is false. Delete $T(e, e, 123)$ from the exception list. The table supplies the complete proof of the stronger corrected scope, and the later claim that only $T(e, 23, e)$ and $T(e, 13, e)$ remain then follows.

Verifiable source: Full paper, version 2

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2410.02032v2
2. Full paper, version 2
3. Cassaigne–Labbé–Leroy input

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.