MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

p-Adically convergent loci in varieties arising from periodic continued fractions

Laura Capuano, Marzio Mula, Lea Terracini, Francesco Veneziano

Paper	arXiv:2402.00739v3
Source version	Submitted 10 July, 2026; v1 submitted 1 February, 2024; announced February 2024.
Audit date	August 15, 2026
Prompt	Standardized prompt
Novelty cutoff	February 1, 2024
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains wrong statements

The necessary-and-sufficient convergence criterion and the principal finiteness and Pell-recurrence results are supported. Proposition 4.9(a), however, incorrectly asserts that the type $(2, 1)$ locus is empty when the discriminant is $4A^2$; an explicit rational point contradicts that assertion. This error does not affect the proposition's finiteness conclusion or its description of the convergent locus.

Theorem 3.1: The periodic convergence criterion is correct

Pages 6–9 · Theorem 3.1 · arXiv:2402.00739v3

Correct

All cyclic period matrices are conjugate and therefore have the same trace and determinant. If the trace has absolute value greater than 1, their eigenvalues have distinct absolute values $|\mu|_p > 1 > |\nu|_p$. Condition (ii) exactly excludes a cyclic period matrix whose dominant eigenline is the wrong coordinate line. The first columns of its powers then converge to the dominant fixed point, and the identities $M_{\ell k+j-1} = M_{j-1}T_j^{\ell}$ make every residue class of convergents approach the same point. Conversely, the equal-modulus and exceptional triangular cases exhibited in the proof prevent convergence, establishing necessity as well as sufficiency.

Proposition 4.9(a): The claimed emptiness at $\Delta = 4A^2$ is false

Page 12 · Proposition 4.9(a) and its proof · arXiv:2402.00739v3

Incorrect

The proposition states that, for $A \neq 0$, $V(F)_{2,1}(\mathcal{O})$ is empty when $\Delta = 4A^2$. Take

$$F(x) = x^2 + 2x,$$

so $A = 1$, $B = 2$, $C = 0$, and $\Delta = 4 = 4A^2$. Substitution into the three defining equations (13) shows directly that

$$(b_1, b_2, a_1) = (-1, 0, 0) \in V(F)_{2,1}(\mathcal{O}).$$

Equivalently, equation (14) at $b_2 = 0$ reduces to $b_1^2 + 2b_1 + 1 = 0$. Thus the printed emptiness conclusion has a formal counterexample. The point has $a_1 = 0$ and is not convergent, so Proposition 4.9(b) is unaffected.

Verifiable source: Full paper, version 3

Propositions 4.18 and Theorem 4.24: The pure-radical classification and finiteness conclusions are correct

Pages 18–24 · Proposition 4.18 and Theorem 4.24 · arXiv:2402.00739v3

Correct

Eliminating the middle partial quotient gives the generalized Pell equation $a_1^2 - da_3^2 = d - 1$ and the identity $a_2 = (d-1)/(a_1 - da_3)$. The convergence criterion forces a_1, a_3 to be p-adic units in $\mathbb{Z}$, and reduction of the second fraction gives exactly the displayed form $a_2 = v/p^s$. Conversely, those arithmetic conditions satisfy both parts of Theorem 3.1. The resulting Pell orbits obey the stated nondegenerate binary recurrence, so the cited perfect-power theorem yields finiteness for fixed p and, when $s > 1$, finiteness as p varies.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The main eigenvalue argument and the Pell-recurrence arguments are correct. The proof of Proposition 4.9(a) divides by b_2^2 without treating $b_2 = 0$, and that omitted case produces an actual counterexample to the proposition's emptiness clause. A verified local repair preserves the advertised finiteness result.

Proof of Proposition 4.9(a): The $b_2 = 0$ branch is lost before the modular obstruction is applied

Page 12 · final paragraph of the proof of Proposition 4.9(a) · arXiv:2402.00739v3

Incorrect as written · verified local repair

For $\Delta = 4A^2$, the discriminant of equation (14), regarded as a quadratic in b_1, is

$$4A^2 b_2^2 (b_2^2 + 2).$$

The proof divides by $4A^2 b_2^2$ and applies a modulo-4 obstruction to $b_2^2 + 2 = v^2$, but it never first establishes $b_2 \neq 0$. The counterexample in Part 1 lies exactly in the omitted branch. Verified repair: split into cases. If $b_2 \neq 0$, the printed clearing-denominators and modulo-4 argument excludes solutions. If $b_2 = 0$, equation (14) has the double solution $b_1 = -B/(2A)$; equations (13) then force $a_1 = 0$. Retain this one point precisely when $-B/(2A) \in \mathcal{O}$. The corrected locus is still finite and its added point is nonconvergent.

Verifiable source: Full paper, version 3

Proof of Theorem 3.1: Dominant-eigenline analysis

Pages 6–9 · proof of Theorem 3.1 · arXiv:2402.00739v3

Correct and complete

The proof treats scalar, non-semisimple, diagonalizable, and coordinate-eigenline cases separately. The unit-determinant relation prevents an unaddressed eigenvalue-size regime, and the cyclic-conjugacy identities propagate the limiting fixed point through all positions in the period. No substantive case is missing.

Proofs of Proposition 4.18 and Theorem 4.24: Integral-point and recurrence reductions

Pages 18–24 · Sections 4.6.1–4.6.2 · arXiv:2402.00739v3

Correct and complete

The projective completion has three points at infinity, so Siegel's theorem applies to the S-integral locus. The valuation argument recovers integral outer partial quotients and the exact denominator power. Each Pell class gives a nondegenerate second-order recurrence with the hypotheses required by the quoted perfect-power theorem; absorbing the fixed prime into the finite set of allowed prime divisors proves the fixed-p case, while taking the varying prime as the powered base proves the $s > 1$ case.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 3

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.