MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

On Nontrivial Winning and Losing Parameters of Schmidt Games

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Paper	arXiv:2401.00614v3
Source version	Submitted 20 February, 2025; v1 submitted 1 January, 2024; announced January 2024.
Audit date	August 19, 2026
Prompt	Standardized prompt
Novelty cutoff	January 1, 2024
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Correct
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Correct

The stated winning and losing regions for digit-frequency sets and the two fixed-constant badly approximable sets are correct. The proof of the losing region for the base-d set omits the rational-$\log_d(\alpha\beta)$ case, and an auxiliary scaling lemma has the sign of a logarithm reversed, but the verified repairs below establish the conclusions without changing their hypotheses.

Theorem 2.2: Digit-frequency winning and losing regions

Pages 4–5 and 13–16 · Theorem 2.2 and its proof · arXiv:2401.00614v3

Correct

For irrational $\log_m(\alpha\beta)$, density of the logarithmic orbit lets a player rescale a game interval to any prescribed length window, after correcting the sign in Lemma 3.5. The subsequent block strategies force arbitrarily long biased binary blocks for the losing assertions and give Alice a uniform supply of the required digits for the winning assertions. The quantitative inequalities in the theorem are exactly those needed by those strategies, so the central frequency conclusions follow after the auxiliary repair.

Theorem 2.5: The base-d fixed-constant badly approximable set is losing in the stated region

Page 7 and pages 16–17 · Theorem 2.5 and its proof · arXiv:2401.00614v3

Correct

Put $r = \alpha\beta$, $\gamma = 1 - 2\alpha + \alpha\beta$, $L = (1-\beta)^{-1}$, and $U = 2c(1-r)/(\beta\gamma)$. The theorem's hypothesis is $U > L$. At a trigger time with $L < d^N|A_t| < U$, Bob can center the next ball at a rational m/d^N; the lower inequality permits that center and the upper inequality, together with the center-control estimate, traps all later play in the corresponding forbidden interval. Such trigger times exist for every r: an irrational logarithmic step gives a dense orbit, while for a rational step Bob chooses the freely available initial logarithmic phase in the projected interval $[\log_d L, \log_d U]$. This supplies the case omitted from the printed proof and verifies the stated theorem.

Theorem 2.6: The binary fixed-constant badly approximable set is winning in the stated region

Page 7 and pages 17–20 · Theorem 2.6 and its proof · arXiv:2401.00614v3

Correct

At each relevant dyadic scale, Alice either keeps the current interval away from the unique forbidden neighborhood or advances to the next scale while preserving the inductive distance bound. The theorem's two parameter inequalities make both alternatives legal. The finite number of smaller scales is immaterial because the target is defined by eventual avoidance. Correcting the two local index and exponent typos identified below makes the invariant agree with the displayed strategy and proves the statement.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

Two material steps are not correct as written. Lemma 3.5 asks for a positive fractional part below a negative number, and the proof of Theorem 2.5 uses equidistribution despite allowing rational logarithmic game parameters. Both steps have verified repairs. The other discrepancies are uniquely determined notation errors and do not affect the paper's overall correctness.

Lemma 3.5: The logarithmic rescaling inequality has a reversed sign

Pages 9–10 · Lemma 3.5 and proof · arXiv:2401.00614v3

Incorrect as written · verified repair

Because $0 < \alpha\beta < 1$, the printed quantity $\log_{\alpha\beta}(1+\varepsilon)$ is negative, so the required inequality for a positive fractional part is impossible. Set $\delta = -\log_{\alpha\beta}(1+\varepsilon) > 0$, $\xi = -\log_{\alpha\beta} m$, and $\chi = \log_{\alpha\beta}(L/l)$. Density of the orbit of ξ modulo one gives k with $\{k\xi + \chi\} < \delta$. Taking $n = \lfloor k\xi + \chi \rfloor$ then gives

$$\frac{L}{m^k} \le (\alpha\beta)^n l < (1+\varepsilon)\frac{L}{m^k},$$

which is the exact rescaling conclusion used later.

Proof of Theorem 2.5: The rational logarithmic-step case is omitted

Pages 16–17 · proof of Theorem 2.5 · arXiv:2401.00614v3

Incomplete as written · verified repair

The proof invokes equidistribution of the sequence of logarithmic interval lengths, but the theorem does not assume $\log_d(\alpha\beta)$ irrational. The repair is to use the freedom in Bob's initial interval length. If the logarithmic step is irrational, the printed density argument applies. If it is rational, Bob chooses the initial phase in the nonempty projected interval $[\log_d L, \log_d U]$ modulo one. The phase then returns periodically, producing infinitely many scales N with $L < d^N |A_t| < U$. The same center choice and trapping argument finish the proof. The printed closed-limsup-to-open-limsup inclusion is also reversed; the direct trapping argument proves open membership and needs no such inclusion.

Proof of Theorem 2.6: A dyadic distance and one induction index are mistyped

Page 19 · proof of Theorem 2.6 · arXiv:2401.00614v3

Typo

When Bob's interval avoids the forbidden neighborhood at scale q_i, the directly obtained distance is $c/2^{q_i-1}$, not $c/2^{q_i+1}$. In the next induction transition, the new error variable is e_{i+1} rather than e_i. The surrounding inequalities use the corrected stronger distance and the next index, so these are uniquely repairable typographical errors.

Auxiliary game notation: Several local symbols have unique contextual corrections

Pages 4 and 9–16 · theorem displays and symmetry lemmas · arXiv:2401.00614v3

Typo

The statement of Theorem 2.2 omits the backslash in one occurrence of α; the invariant in Lemma 3.3 should read $m^k S = m^k + S = S$; and a symmetry sentence reverses membership when passing between S and $-S$. Each intended symbol is fixed by the adjacent definition and none changes a parameter range or a strategic step.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:2401.00614v3

arXiv acknowledgement

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small $delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every $delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.