MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Anti-classification results for weakly mixing diffeomorphisms

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Paper	arXiv:2303.12900v2
Source version	Submitted 4 August, 2026; v1 submitted 22 March, 2023; announced March 2023.
Audit date	August 18, 2026
Prompt	Standardized prompt
Novelty cutoff	March 22, 2023
Paper license	CC BY 4.0
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains wrong statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains wrong statements

Theorem A is false in its assertion that every equivalence relation intermediate between isomorphism and Kakutani equivalence gives a complete analytic set: arbitrary intermediate relations need not be analytic. The reduction does verify the universal non-Borel conclusion, and it gives complete analyticity for the specifically analytic relations of isomorphism and Kakutani equivalence. The analogous unrestricted clauses in Theorems B and C and the asserted injectivity in Theorem 3 are not verified.

Theorem A: An arbitrary intermediate equivalence relation need not be analytic

Page 3 and pages 5-6 · Theorem A and the reduction argument · arXiv:2303.12900v2

Incorrect

The exact claim is that for every equivalence relation R with $R_{\text{iso}} \subseteq R \subseteq R_{\text{Kak}}$, the restriction of R to pairs of weakly mixing transformations is complete analytic. This is false because no definability hypothesis is imposed on R. For a formal counterexample, choose a continuum family $(B_h)_{0<h<\log 2}$ of pairwise nonisomorphic Bernoulli shifts indexed injectively by entropy. Every B_h is mixing, and all finite-positive-entropy Bernoulli shifts are Kakutani equivalent. Fix h_0. For every set $A \subseteq (0, \log 2)$ containing h_0, let R_A merge precisely the isomorphism classes of the systems B_h with $h \in A$ into one class and leave every other isomorphism class unchanged. Then $R_{\text{iso}} \subseteq R_A \subseteq R_{\text{Kak}}$, and the restrictions of the R_A to weakly mixing pairs are distinct for $2^{\mathfrak{c}}$ choices of A. A Polish space has only $\mathfrak{c}$ analytic subsets, so at least one such restriction is not analytic and therefore cannot be complete analytic. The paper's reduction nevertheless proves that every one of these restrictions is Σ^1_1-hard and hence non-Borel.

Verifiable source: Ornstein-Rudolph-Weiss, Equivalence of measure preserving transformations

Theorems A-C for isomorphism and Kakutani equivalence: The named classification relations are complete analytic on the constructed classes

Pages 3-6 and pages 50-52 · Theorems A-C and Theorem 3 · arXiv:2303.12900v2

Correct

For $R = R_{\text{iso}}$ and $R = R_{\text{Kak}}$, the relevant pair relations are analytic. The constructed continuous map sends a tree $\mathcal{T}$ to a weakly mixing transformation $T = \Phi(\mathcal{T})$, and Sections 6.2-6.3 prove that $\mathcal{T}$ is ill-founded exactly when T and T^{-1} are R-equivalent. Composing with $T \mapsto (T, T^{-1})$ reduces the complete analytic set of ill-founded trees to each pair relation. Analyticity plus this reduction proves complete analyticity; the same reasoning applies after the smooth and real-analytic realization maps used in Theorems B and C.

Verifiable source: Full paper, version 2

Theorems B and C for arbitrary intermediate relations: The complete-analytic clauses lack an analyticity hypothesis

Pages 3-6 and page 52 · Theorems B-C and their proofs · arXiv:2303.12900v2

Not able to verify

As stated, these theorems quantify over every equivalence relation R between the restrictions of isomorphism and Kakutani equivalence, without requiring R to be analytic. The proofs establish a continuous reduction from ill-founded trees and therefore establish Σ^1_1-hardness and non-Borelness. They do not establish that the target is analytic, which is an independent requirement in the paper's own definition of complete analytic. The abstract counterexample above proves the analogous clause of Theorem A false, but the present audit does not supply the additional smooth or real-analytic realization family needed to turn it into a counterexample in each of these two narrower ambient spaces. Thus their complete-analytic clauses are not verified; their non-Borel conclusions are verified.

Verifiable source: Full paper, version 2

Theorem 3: The reduction properties are correct

Pages 5-6 and pages 50-52 · Theorem 3 and Section 6 · arXiv:2303.12900v2

Correct

Apart from the separate injectivity clause below, the asserted properties are established. Lemma 64 proves continuity; Proposition 31 together with the construction requirements proves weak mixing; Lemmas 65-66 turn an infinite branch into an isomorphism with the inverse; and Section 6.3 transfers the spacer-ignored separation estimates to prove that Kakutani equivalence with the inverse forces an infinite branch. These implications are sufficient for the continuous reduction used in Theorems A-C.

Verifiable source: Full paper, version 2

Theorem 3, injectivity clause: The claim that Φ is one-to-one is not established

Pages 5-6 and pages 50-52 · Theorem 3 and Section 6 · arXiv:2303.12900v2

Not able to verify

Theorem 3 calls $\Phi : \text{Trees} \to \text{Diff}^\infty_\lambda(M)$ one-to-one. Section 6 proves continuity and the three dynamical implications needed for the reduction, but it contains no argument that two distinct trees produce distinct diffeomorphisms. The prefix dependence used in Lemma 64 proves continuity only; it does not give a converse separation statement. A verification would require a quantitative or exact construction-level argument showing that the first differing tree datum survives all subsequent conjugations and the realization map. No such argument or independent proof was found. Injectivity is not needed for the reduction or the non-Borel conclusions.

Verifiable source: Full paper, version 2

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The proofs of Theorems A-C establish hardness and non-Borelness but incorrectly treat hardness as complete analyticity without proving that an arbitrary intermediate relation is analytic. The proof of Theorem 3 omits its injectivity clause. The central weak-mixing, realization, branch-isomorphism, and non-Kakutani chains are otherwise correct. Three groups of local notation defects have unique harmless corrections.

Proofs of Theorems A-C: A reduction proves analytic hardness, not analytic membership

Pages 5-6 and page 52 · deductions from Theorem 3 · arXiv:2303.12900v2

Incorrect as written

The proof constructs a continuous reduction of the complete analytic set of ill-founded trees to the target pair set. This proves that the target is Σ^1_1-hard. It does not prove that the target itself is analytic, whereas Definition 2 requires both analyticity and universal reducibility for the label complete analytic. For arbitrary intermediate R, analyticity does not follow from $R_{\mathrm{iso}} \subseteq R \subseteq R_{\mathrm{Kak}}$; Theorem A is formally contradicted by the family in Part 1. Downstream dependency: this is the sole step claimed to establish complete analyticity in Theorems A-C. Verified repair: for unrestricted R, replace complete analytic by Σ^1_1-hard and retain the proved non-Borel conclusion; alternatively, add the hypothesis that the relevant restriction of R is analytic, under which the existing reduction does prove complete analyticity.

Verifiable source: Full paper, version 2

Proof of Theorem 3: Injectivity of the tree-to-diffeomorphism map is not proved

Pages 50-52 · Section 6 · arXiv:2303.12900v2

Incomplete as written

Section 6 announces verification of all properties required by Theorem 3, but Lemma 64 proves only continuity and the remaining subsections prove weak mixing and the two branch/Kakutani implications. None proves that $\Phi(\mathcal{T}) = \Phi(\mathcal{S})$ forces $\mathcal{T} = \mathcal{S}$. Downstream dependency: this leaves the literal one-to-one clause of Theorem 3 unsupported, although no later reduction needs injectivity. Repair classification: No repair supplied; a new separation argument for distinct construction prefixes is required.

Verifiable source: Full paper, version 2

Central construction and dynamical implications: The substantive reduction chain is correct and complete

Pages 18-23 and pages 30-52 · Propositions 28 and 31; Sections 4.3, 5, and 6 · arXiv:2303.12900v2

Correct and complete

Requirement (R4) and the counting in Proposition 31 verify the weak-mixing criterion at the stated mixing times. The substitution construction enforces the required uniformity and separation, and the symbolic systems are transferred by the smooth realization theorem and its real-analytic counterpart. For an infinite branch, the compatible odd-parity actions in Lemmas 65-66 give an isomorphism with the reversed system. For a well-founded tree, Lemmas 62-63 supply the same $\bar{f}$ separation estimates used in the cited Gerber-Kunde argument; that argument explicitly ignores the spacer symbols, so the change from the circular to the twisting operator preserves the non-Kakutani conclusion. No substantive defect was found in these chains.

Verifiable source: Gerber-Kunde, Non-classifiability of Ergodic Flows up to Time Change

Preliminaries: The join of two partitions repeats the wrong partition name

Page 8 · definition of $P \vee Q$ · arXiv:2303.12900v2

Typo

Printed: $P \vee Q = \{c \cap d : c \in P, d \in P\}$. Correction: replace the second P by Q. The surrounding sentence defines the join of P and Q, so the intended correction is unique and no later argument changes.

Verifiable source: Full paper, version 2

Definition 20: A twisted system is said to come from the wrong construction type

Page 14 · Definition 20 · arXiv:2303.12900v2

Typo

Printed: a system built from a circular construction sequence is called a twisted system. Correction: replace circular by twisted. Definition 19 has just defined twisted construction sequences, and every subsequent use of K^{twist} uses that construction, so this is a unique harmless terminology correction.

Verifiable source: Full paper, version 2

Requirements (R3) and proof of Proposition 31: Two terminal indices lie one step beyond their declared range

Pages 21-23 · requirement (R3) and the case distinction in the proof of Proposition 31 · arXiv:2303.12900v2

Typo

The tuples $b_n(i,s)$ are defined only for $0 \leq i < 2^{n+2}q_n$. Requirement (R3) nevertheless ends its displayed list at $b_n(2^{n+2}q_n, s)$; the explanatory sentence immediately below ends it correctly at $2^{n+2}q_n - 1$. Later, the proof of Proposition 31 treats the boundary case $i_2 = 2^{n+2}q_n$, although its standing range is $0 \leq i_2 < 2^{n+2}q_n$. In both places the unique correction is to replace the terminal index by $2^{n+2}q_n - 1$. The earlier rotation formula (4.16), the cyclic successor, and the subsequent counting all use that corrected endpoint, so the argument is unchanged.

Verifiable source: Full paper, version 2

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Full paper, version 2
2. Ornstein-Rudolph-Weiss, Equivalence of measure preserving transformations
3. Gerber-Kunde, Non-classifiability of Ergodic Flows up to Time Change

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\text{delta})$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(\text{delta})$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.