

# Correctness, proof completeness, and novelty audit

## Abundance of Dirichlet-improvable pairs with respect to arbitrary norms

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|                |                                                                              |
|----------------|------------------------------------------------------------------------------|
| Paper          | arXiv:2107.10298v2                                                           |
| Source version | Submitted 19 October, 2021; v1 submitted 21 July, 2021; announced July 2021. |
| Audit date     | August 19, 2026                                                              |
| Prompt         | Standardized prompt                                                          |
| Novelty cutoff | July 21, 2021                                                                |
| Paper license  | CC BY 4.0                                                                    |
| Scope          | Statements, proofs, and novelty only                                         |

**Important limitation.** AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

### SUMMARY

|    |            |                         |
|----|------------|-------------------------|
| 01 | Statements | Correct                 |
| 02 | Proofs     | Correct                 |
| 03 | Novelty    | No non-novelty findings |

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## 01 / STATEMENTS

# Statements

## Correct

For every norm on the plane, badly approximable pairs are correctly shown to be Dirichlet-improvable, hence the improvable set is hyperplane absolute winning; the top of the associated Dirichlet spectrum is an accumulation point. One displayed subscript is a harmless typo.

## Theorem 1.1: Badly approximable pairs are Dirichlet-improvable for every norm

Page 2 and Section 5 · main theorem and completion · arXiv:2107.10298v2

## Correct

The Dani correspondence embeds a pair into the space of unimodular three-lattices and identifies strict improvement with eventual avoidance of the critical locus of the cylinder norm  $\eta(x_1, x_2, x_3) = \max\{\nu(x_1, x_2), |x_3|\}$ . The Chalk–Rogers–Mahler classification places each critical lattice in a structured family. If a badly approximable orbit accumulated on that family, the displayed lattice bases would produce integer vectors violating its uniform lower bound. Thus the orbit eventually avoids the critical locus.

## Corollary 1.2: The top of the Dirichlet spectrum is not isolated

Page 2 and Section 5 · spectrum corollary · arXiv:2107.10298v2

## Correct

The main theorem gives a hyperplane-absolute-winning set below the top value. The cited dimension-drop theorem bounds the Hausdorff dimension of vectors whose orbit avoids a fixed neighborhood of the critical locus. If the top value were isolated, all vectors in the winning set would avoid one common neighborhood, contradicting full Hausdorff dimension of a winning set.

## 02 / PROOFS

# Proofs

## Correct

The dynamical reformulation, cylinder critical-locus classification, orbit-avoidance argument, and dimension-drop contradiction are correct and complete. Two local norm subscripts have uniquely determined corrections.

## Sections 2–4: The critical-locus alternatives exhaust all cylinder lattices

Pages 3–13 · dynamical proposition and critical-locus theorem · arXiv:2107.10298v2

### Correct and complete

The critical radius normalization gives  $c_\nu = r_\eta^3$ . The Chalk–Rogers alternatives are translated into unimodular-lattice coordinates without losing boundary cases, and Mahler’s reduction handles the exceptional planar section. Each resulting family is then checked against the diagonal action used by the Dani correspondence.

## Definition of the supremum-norm improvable set: The formula uses the Euclidean spectrum function

Page 2 · display defining the supremum-norm improvable set · arXiv:2107.10298v2

### Typo

The set is denoted  $\mathbf{DI}_\infty$  and the surrounding paragraph discusses  $c_\infty$ , but the display reads  $c_2(\mathbf{x}) < 1$ . The uniquely consistent correction is  $c_\infty(\mathbf{x}) < 1$ ; all later definitions use the norm-matched function  $c_\nu$ .

## Proof of Theorem 1.1: The limiting lattice is assigned to the wrong critical locus

Page 14 · first paragraph of the proof of Theorem 1.1 · arXiv:2107.10298v2

### Typo

The proof assumes that  $a_{s_k}\Lambda_{\mathbf{x}}$  converges to a lattice in the critical locus of the three-dimensional cylinder norm  $\eta$ , but the displayed membership is  $\Lambda \in \mathcal{L}_\nu$ . Proposition 5.1 and the ambient space  $X_3$  force the correction  $\Lambda \in \mathcal{L}_\eta$ ; the subsequent two-case argument already uses exactly that locus.

## Section 5: Bad approximation excludes every critical-locus accumulation case

Pages 14–15 · proof of the main theorem · arXiv:2107.10298v2

### Correct and complete

In each structural alternative, convergence to the critical locus supplies primitive integer vectors whose first two coordinates and denominator violate the compact-orbit lower bound for a badly approximable pair. The estimates are uniform along the convergent subsequence, and the strict inequality in the improvable definition follows from compactness of the avoided critical locus.

03 / NOVELTY

# Novelty

No non-novelty findings

No non-novelty findings.

**METHOD**

# Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

## Sources checked

1. Source paper: [arXiv:2107.10298v2](https://arxiv.org/abs/2107.10298v2)

## arXiv acknowledgement

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## APPENDIX

# Standardized prompt

*You are a mathematical auditor. Evaluate only the correctness of the paper’s central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.*

## PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

*Identify the paper’s central statements and assess each one separately using exactly one of these outcomes:*

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

*For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.*

*Assign the overall statements status by the following priority:*

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

## PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

*Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:*

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

*Do not flag omitted routine details, compressed arguments, or “analogously” steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:*

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

*Assign the overall proofs status by the following priority:*

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

## PART 3 — NOVELTY OF THE MAIN STATEMENTS

*This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.*

*For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.*

*Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level “no prior source found” entries. If the list is empty, the section may say only “No non-novelty findings.”*

## PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. *Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.*

2. *Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.*

3. *Immediate-consequence closure*: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if  $P(\delta)$  is proved for every sufficiently small  $\delta > 0$  and  $P$  is immediately monotone in the required direction, then the claim that  $P(\delta)$  holds for every  $\delta > 0$  requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for  $t > 0$  and subsequently applied at  $t = 0$  remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as *Typo*, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as *Minor formal correction*, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be *Correct in Part 1* because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an *Incorrect theorem classification*.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

## END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.