MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Almost elementary étale groupoids

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Paper	arXiv:2011.01182v3
Source version	Submitted 10 August, 2026; v1 submitted 2 November, 2020; announced November 2020.
Audit date	August 18, 2026
Prompt	Standardized prompt
Novelty cutoff	November 2, 2020 (all audited central results already appear in arXiv v1)
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

The equivalences with Matui almost finiteness, Kerr almost finiteness, and the characterization by almost elementariness in measure plus groupoid strict comparison are correct; the two defects in their printed proof chains have complete repairs. The central claim that every minimal almost elementary groupoid has a tracially $\mathcal{Z}$-stable reduced groupoid C^*-algebra is not able to be verified because its required nesting proposition contains false inclusions and an impossible measure estimate, and no complete replacement argument was found. The amenable classification corollary inherits that unresolved dependency. No counterexample to either unsupported statement is asserted.

Theorem 8.13: Almost elementariness is characterized by measure approximation and comparison

Page 45 · Theorem 8.13 and Lemma 8.12 · arXiv:2011.01182v3

Correct

The finite-unit case follows from the cited finite-principal characterization. For an infinite unit space, almost elementariness gives almost elementariness in measure and strict comparison through Propositions 8.9 and 8.11. Conversely, if $M(\mathcal{G}) \neq \varnothing$, the uniform positive lower bound for every nonempty open target lets strict comparison absorb the measure-small remainder; if $M(\mathcal{G}) = \varnothing$, the comparison hypothesis is already vacuous in the required direction. Lemma 8.12's printed semicontinuity inference is false, but the finite-bisection argument given in the proof findings supplies the needed uniform bound and verifies the theorem.

Verifiable source: Almost elementary étale groupoids, version 3

Theorem 10.6: Matui almost finiteness has the stated two-property characterization

Pages 58–60 · Theorem 10.6 · arXiv:2011.01182v3

Correct

A compact open elementary subgroupoid decomposes into a compact open castle, and a finite clopen refinement supplies the required fineness without changing the fibers. In the converse direction, compact-open subequivalence moves the remainder into a small set of castle levels; refining along the corresponding ranges and adjoining the pulled-back source levels produces an elementary subgroupoid with full unit space. If $N = \sup_x |Kx|$, the choices $\delta < \varepsilon/(2N)$ and $|S_{\mathcal{T}}| \leq \delta|\mathcal{T}^{(0)}|$ give the required K-boundary estimate. The harmless empty-K case follows after adjoining the unit space to K.

Verifiable source: Almost elementary étale groupoids, version 3

Theorem 11.5: Kerr almost finiteness has the stated transformation-groupoid characterization

Pages 61–63 · Theorem 11.5 · arXiv:2011.01182v3

Correct

From groupoid castles refining the canonical partition, one recovers dynamical castles with the same level counts, comparison relation, and Følner estimate. Conversely, for an auxiliary tolerance δ chosen sufficiently smaller than the requested ε, the cores $T_i = \bigcap_{\gamma \in F} \gamma^{-1} S_i$ satisfy $|S_i \setminus T_i| \leq |F|\delta|S_i|$ and remain large. Allocating disjoint target blocks inside T_i for both the original comparison range and the discarded boundary levels gives the relative-remainder comparison; the complete bookkeeping appears in the proof finding below. The cited fiberwise-amenability result also verifies that a transformation groupoid is fiberwise amenable exactly when the acting group is amenable.

Verifiable source: Ma–Wu, Fiberwise amenability of étale groupoids

Theorem 13.11: Tracial $\mathcal{Z}$-stability is not established by the available proof

Pages 72–74 and 81–83 · Proposition 12.11 and Theorem 13.11 · arXiv:2011.01182v3

Not able to verify

The theorem requires Proposition 12.11 to produce a faithfully nested castle with arbitrarily large minimal multiplicity while retaining the required small remainder. Equations (12.1)–(12.2) and the subsequent measure expansion used to prove that proposition are formally unsupported and, for permitted parameter choices, contradictory to the probability bound, as detailed in the proofs findings. The remainder of Theorem 13.11 correctly turns such a nested castle into an approximately central order-zero map with Cuntz-small complement, but that conditional construction does not supply the missing castle. No counterexample to tracial $\mathcal{Z}$-stability is obtained, so the supported outcome is Not able to verify rather than Incorrect.

Verifiable source: Almost elementary étale groupoids, version 3

Corollary 13.13: The amenable classification conclusions inherit the unresolved main input

Pages 83–84 · Corollary 13.13 · arXiv:2011.01182v3

Not able to verify

For an infinite unit space, the proof obtains simplicity and tracial $\mathcal{Z}$-stability only through Theorem 13.11. Conditional on that input, amenability gives nuclearity and the UCT, Hirshberg–Orovitz gives $\mathcal{Z}$-stability, and the cited nuclear-dimension, classification, quasidiagonality, and pure-infiniteness implications apply. Because the required tracial-$\mathcal{Z}$ input is not verified here, the full corollary is not verified. This is a dependency finding, not evidence that any conclusion is false.

Verifiable source: Almost elementary étale groupoids, version 3

Corollary 13.13: The nuclear-dimension conclusion should be an upper bound

Pages 83–84 · statement and proof of Corollary 13.13 · arXiv:2011.01182v3

Minor formal correction

The corollary allows a finite unit space but prints $\dim_{\mathrm{nuc}}(C_r^*(\mathcal{G})) = 1$. For a finite minimal almost elementary groupoid, the reduced algebra is a matrix algebra and has nuclear dimension 0. Replace the displayed equality by $\dim_{\mathrm{nuc}}(C_r^*(\mathcal{G})) \leq 1$. This is the bound supplied by the cited theorem, covers both finite and infinite cases, and leaves every classification consequence unchanged. Repair classification: Verified local repair.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

Lemma 8.12 uses upper semicontinuity in the wrong direction but has a complete finite-bisection repair. Theorem 11.5 omits the discarded boundary levels from its remainder and therefore prints a false set equality; a smaller auxiliary tolerance and two disjoint target blocks completely repair that equivalence. Proposition 12.11 has the decisive unresolved defect: its selected ranges need not contain the deep core, and its asserted $N|\mathcal{E}_-|$ measure expansion can exceed one. Theorem 13.11 consequently remains incomplete. Three literal notation or indexing errors have unique harmless corrections and are reported in yellow.

Lemma 8.12: Upper semicontinuity does not give the claimed positive minimum

Page 45 · proof of Lemma 8.12 · arXiv:2011.01182v3

Incorrect as written · verified repair

The proof says that $\mu \mapsto \mu(\overline{O'})$ is upper semicontinuous and therefore has a minimum on $M(\mathcal{G})$. Upper semicontinuity on a compact space guarantees a maximum, not a minimum; pointwise positivity alone does not exclude infimum zero. A complete repair uses minimality and compactness directly. For every unit u, choose an open bisection B_u with $u \in s(B_u)$ and $r(B_u) \subseteq O$. A finite family $B_1, \ldots, B_m$ has sources covering $\mathcal{G}^{(0)}$. Invariance then gives

$$1 \leq \sum_{j=1}^{m} \mu(s(B_j)) = \sum_{j=1}^{m} \mu(r(B_j)) \leq m\mu(O),$$

so $\mu(O) \geq 1/m$ uniformly. This repairs every use in Theorem 8.13.

Theorem 10.6: The compact-open castle argument is complete

Pages 58–60 · proof of Theorem 10.6 · arXiv:2011.01182v3

Correct and complete

The elementary subgroupoid decomposition, common clopen refinement, compact-open realization of the remainder comparison, induced enlargement of each tower, and final fiber-count estimate all preserve the stated hypotheses. Writing $N = \sup_x |Kx|$ gives

$$|K(\mathcal{E})u| \leq (1+\delta)|\mathcal{T}^{(0)}| + N\delta|\mathcal{T}^{(0)}| \leq (1+\varepsilon)|(\mathcal{E})u|$$

after the printed choice $\delta < \varepsilon/(2N)$ when $N \geq 1$; adjoining units handles the remaining case.

Theorem 10.6, only-if direction: The refined object is D, not C

Page 59 · first paragraph of the proof of Theorem 10.6 · arXiv:2011.01182v3

Typo

After constructing the refined castle D, the proof says that if $D \cap L \neq \varnothing$ then some cover element meets C and therefore $C \subseteq V$. Replace both occurrences of C in that sentence by D. The common clopen partition was constructed precisely so that each D is either contained in or disjoint from each V, making the correction unique and harmless.

Theorem 11.5, reverse direction: The printed remainder equality drops all discarded boundary levels

Pages 62–63 · final two paragraphs of the proof of Theorem 11.5 · arXiv:2011.01182v3

Incorrect as written · verified repair

The groupoid castle $\mathcal{C}$ is built from $T_i = \bigcap_{\gamma \in F} \gamma^{-1} S_i$, but the proof prints

$$\mathcal{G}^{(0)} \setminus \bigcup \mathcal{C}^{(0)} = X \setminus \bigsqcup_i S_i V_i.$$

The left side is instead $X \setminus \bigsqcup_i T_i V_i$ and also contains the boundary levels $(S_i \setminus T_i)V_i$. For a verified repair, choose $\eta > 0$ with $2\eta < \varepsilon(1-\eta)$ and invoke almost finiteness with δ satisfying $|F^{-1}F|\delta < \eta$ and $\delta < \eta$. Then $|S_i \setminus T_i| \leq \eta |S_i|$, $|S_i'| < \eta |S_i|$, and $|T_i| \geq (1-\eta)|S_i|$. Inside each T_i, reserve two disjoint blocks totaling fewer than $2\eta|S_i| < \varepsilon |T_i|$ levels. Lemma 8.5 moves the original comparison range into one block and the discarded boundary into the other; composing and combining these disjoint comparisons gives the required relative-remainder control. The extendability argument is unchanged.

Theorem 11.5, tolerance notation: The two halves use mismatched tolerance symbols

Pages 61–63 · proof of Theorem 11.5 · arXiv:2011.01182v3

Typo

In the first half, the input tolerance is δ but conditions (iii)–(iv) and the subsequent level counts use ε; replace those occurrences by δ. In the second half, the displayed choice $\delta < \varepsilon/(1-\varepsilon)$ is used to infer $\delta|S_i| \leq \varepsilon|T_i|$ from $|T_i| \geq (1-\varepsilon)|S_i|$; the required factor is $\delta < \varepsilon(1-\varepsilon)$, after the harmless initial reduction $0 < \varepsilon < 1$. Both corrections are mechanically forced by the adjacent inequalities. The more substantive omitted-boundary repair is reported separately.

Proposition 12.11: The selected-range inclusions and measure expansion are invalid

Pages 73–74 · Equations (12.1)–(12.2) and the following measure estimate · arXiv:2011.01182v3

Incorrect as written

For each deep level $D_{+,0}$ and each $E_- \in \mathcal{E}_-^{(0)}$, the construction selects at least N ladders whose ranges lie in E_-. Hence $r(\mathcal{X}) \subseteq \bigcup \mathcal{E}_-^{(0)}$, but Equation (12.1) additionally asserts

$$(\mathcal{D}_+^{-KL_2K_0})^{(0)} \subseteq r(\mathcal{X})$$

without any mechanism placing the source core among those ranges; Equation (12.2) repeats the unsupported inclusion for the inner castle. More decisively, the proof claims

$$\mu\Big(\bigcup r(\mathcal{X}_-)\Big) \geq N|\mathcal{E}_-|\, \mu\Big(\bigcup (\mathcal{D}_-^{-KL_2K_0})^{(0)}\Big).$$

The selected ranges for different core levels can coincide, so the per-source cardinality bound cannot be summed as a globally disjoint expansion. In the permitted case $M(\mathcal{G}) \neq \varnothing$, the contradiction is quantitative: choose O_0 with $\sup_\mu \mu(O_0) < 1/2$. Condition (iii) then gives every core measure greater than $1/2$, while $N > 1$ and $|\mathcal{E}_-| \geq 1$ make the asserted right side exceed 1, although the left side is a probability. This is exactly the estimate used to prove the small remainder in (iv). Repair classification: No repair supplied; a new globally disjoint selection or a different recurrence argument is required.

Lemma 13.10: The double sums must be restricted to nonempty reductions

Pages 80–81 · proof of Lemma 13.10, Equations (13.1)–(13.2) · arXiv:2011.01182v3

Typo

Minimal multiplicity gives $|\operatorname{Red}(\mathcal{T}, D)| \geq nN$ only when $\operatorname{Red}(\mathcal{T}, D) \neq \varnothing$, not for every pair $(\mathcal{T}, D)$. Restrict the two double sums and the preceding universal sentence to the intersecting pairs. The correction is uniquely dictated by Definition 12.9. For those pairs, the reductions are disjoint subsets of $\mathcal{C}^{(0)}$, so

$$\sum nN\,\mu(C_{\mathcal{T}}) \leq \mu\Big(\bigcup \mathcal{C}^{(0)}\Big) \leq 1,$$

and the printed $1/N$ defect bound follows unchanged.

Theorem 13.11: The order-zero construction is conditional on an unproved castle input

Pages 81–83 · proof of Theorem 13.11 · arXiv:2011.01182v3

Incomplete as written

After Proposition 12.11 is invoked, the proof correctly uses Lemmas 13.9–13.10 to construct the matrix embedding, the flat cutoff to obtain approximate centrality, and groupoid strict comparison to make the complement Cuntz-subequivalent to the prescribed positive element. But Proposition 12.11 has not established the faithfully nested castle with both high multiplicity and the small remainder, and that input is used twice: in the trace-defect estimate for Q_2 and in the support control of $1 - \psi(1)$. Because the missing recurrence-and-remainder step is nontrivial and no verified replacement is available, the main proof remains incomplete.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Almost elementary étale groupoids, version 3
2. Ma–Wu, Fiberwise amenability of étale groupoids

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.
- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.
- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.
2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.
3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.
- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.
- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.
- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.
- Plausible repair only: describe the possible repair but do not treat it as established.
- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.
2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.
3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.