MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Hausdorff dimensions of very well intrinsically approximable subsets of quadratic hypersurfaces

Lior Fishman, Keith Merrill, David Simmons

Paper	arXiv:1502.07648v3
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Prompt	Standardized prompt
Novelty cutoff	February 26, 2015
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A green 'no issues found' label may coexist with yellow suspected typos or minor formal issues that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	No statement issues found
02	Proofs	Suspected incomplete or incorrect proofs
03	Novelty	No novelty concerns found

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01 / STATEMENTS

Statements

No statement issues found

The Hausdorff-dimension upper bound for very well intrinsically approximable points on quadratic hypersurfaces is correct. Several defects in the printed neighborhood estimates admit a complete verified repair without changing the theorem's hypotheses or conclusion.

Theorem 1.2: The intrinsic Pollington–Velani bound holds

Pages 2 and 4–9 · Theorem 1.2 and Section 3 · arXiv:1502.07648v3

No issue found

Let $\beta < \alpha$. After the repaired quadratic-neighborhood lemma, every ambient hyperplane section satisfies $\nu(B \cap \mathcal{N}(\mathcal{L} \cap Z, \varepsilon\rho)) \ll \varepsilon^{\beta}\nu(B)$. The intrinsic simplex lemma places rational points of denominator at most κ/ρ_n in one affine hyperplane on each ball of radius ρ_n. For an approximant with exponent $c' > c$, minimality of n gives $q > (\kappa/2)\rho_n^{-1}$, hence $q^{-c'} \ll \rho_n^{c'} \le \rho_n^{c}$ for large n. A maximal ρ_n^c-separated cover then has s-cost bounded by

$$\sum_n \rho_n^{c(s-\delta)+\beta(c-1)},$$

which converges whenever $s > \delta - \beta(1 - 1/c)$. Letting $\beta \uparrow \alpha$ proves the displayed bound $\delta - \alpha(1 - 1/c)$.

Verifiable source: Exact arXiv version 3

02 / PROOFS

Proofs

Suspected incomplete or incorrect proofs

The final Hausdorff–Cantelli argument is sound once its inputs are repaired, but the printed proofs of Lemmas 3.4 and 3.5 contain false inclusions and omit necessary covering cases. The repairs below are complete and preserve all stated constants up to the paper's coarse multiplicative notation.

Lemma 3.4: The tangent-plane estimate is applied to points outside the graph domain

Page 5 · proof of equation (3.1) in Lemma 3.4 · arXiv:1502.07648v3

Suspected incorrect as written · an apparently sound repair

The maximal separated set A is chosen in the ambient set $\mathcal{N}(B\cap\Gamma,\varepsilon\rho)$, but the proof then treats $y,z\in\mathbb{R}^k$ as points of the graph domain $S\subseteq\mathbb{R}^{k-1}$, writes $f(y)$ and $f(z)$, and invokes a path in S joining them. Moreover, y need not lie in $\operatorname{Supp}(\mu)$, so the absolute-decay estimate for $B(y,\widetilde{\rho})$ is not available. Finally, the assertion that all of $B\cap\Gamma$ lies near the tangent plane at one local center is false; only the corresponding local piece can do so. For a verified repair, put $E=\mathcal{N}(B\cap\Gamma,\varepsilon\rho)\cap\operatorname{Supp}(\mu)$ and take $A\subseteq E$ maximal $\widetilde{\rho}$-separated, where $\widetilde{\rho}=\sqrt{\varepsilon}\rho/2$. For each $y\in A$ and each measured point $w\in B(y,\widetilde{\rho})\cap E$, choose graph points $(u,f(u)),(v,f(v))\in B\cap\Gamma$ within $\varepsilon\rho$ of y,w. Their domain coordinates are $O(\widetilde{\rho})$ apart. The K_1-quasiconvex path from u to v and the bound $\|f''\|\le K_2/\rho$ put the local graph piece within $O(\sqrt{\varepsilon}\,\widetilde{\rho})$ of the tangent hyperplane at $(u,f(u))$; adding the two endpoint errors has the same order. Absolute α-decay at the support center y, bounded overlap, and the printed iteration then give (3.1) and (3.2), with only the implicit constant changed.

Lemma 3.5: Signed permutations of one dominant-coordinate patch do not cover a general diagonal quadric

Page 6 · paragraph defining $\widetilde{Z}$ in the proof of Lemma 3.5 · arXiv:1502.07648v3

Suspected incorrect as written · an apparently sound repair

After diagonalizing $P(x)=\sum_i c_i x_i^2-b$, the proof claims $Z=G(\widetilde{Z})$ for the signed permutation group G. This is false when the coefficients c_i are unequal, because permuting coordinates need not preserve Z. Replace this assertion by the finite cover indexed by an orthant and a dominant coordinate j:

$$Z_{\sigma,j}=\{x\in Z:\operatorname{sgn}(x)=\sigma,\ |c_ix_i|\le|c_jx_j|\text{ for all }i\}.$$

Every point belongs to such a patch by choosing j maximizing $|c_jx_j|$. On each patch solve for the coordinate x_j with its fixed sign. The derivative calculation printed for $j=k$ is unchanged after relabeling and gives $|f_i'|\le1$ and $|f_{i\ell}''|\ll1/\min_m|x_m|$. There are only 2^kk patches, so the uniform estimate survives the finite union.

Lemma 3.5: The dyadic shells neither cover the far-from-axis part nor justify the displayed covering inequality

Page 7 · dyadic-shell argument in the proof of Lemma 3.5 · arXiv:1502.07648v3

Suspected incomplete as written · an apparently sound repair

The shells $R_n = \{\rho_{n+1} \le \min_i x_i \le \rho_n\}$ for $n \ge 0$ omit points with $\min_i x_i > \rho$. Also, a maximal separated set in $B \cap R_n \cap \mathrm{Supp}(\mu)$ need not cover the measured set $B \cap \mathcal{N}(\Gamma(S_n, f), \varepsilon\rho)$, whose support points can lie outside R_n. A complete repair is as follows. First handle the portion with $\min_i x_i > \rho$ as one bounded local rectangle around B: there $\|f''\| \ll 1/\rho$, so Lemma 3.4 directly gives $O(\varepsilon^\beta \mu(B))$. For shells satisfying $\rho_n \ge \varepsilon\rho$, choose a maximal ρ_n-separated set in $B \cap \mathcal{N}(\Gamma(S_n, f), \varepsilon\rho) \cap \mathrm{Supp}(\mu)$. These centers are $O(\rho_n)$ from the union V of coordinate hyperplanes; Lemma 3.4, bounded overlap, and absolute decay therefore give

$$\mu(E_n) \ll (\varepsilon\rho/\rho_n)^\beta (\rho_n/\rho)^\alpha \mu(B) = \varepsilon^\beta 2^{-n(\alpha-\beta)} \mu(B).$$

The sum converges because $\beta < \alpha$. The remaining tail $\rho_n < \varepsilon\rho$ lies, after the outer neighborhood is added, in $\mathcal{N}(V, O(\varepsilon\rho))$ and has measure $O(\varepsilon^\alpha \mu(B))$, hence $O(\varepsilon^\beta \mu(B))$. Coordinate-hyperplane points themselves are included in the same last estimate. This proves Lemma 3.5 with the stated uniformity.

Corollary 3.7: The ball radius and neighborhood scale use different variable names

Page 8 · statement of Corollary 3.7 · arXiv:1502.07648v3

Suspected typo

The ball is written $B = B(x, r)$ but the neighborhood thickness is $\varepsilon\rho$. Replace r by ρ in the ball, or replace $\varepsilon\rho$ by εr. The proof and the application on page 9 use one common radius, so the correction is uniquely determined and harmless.

Proof of Theorem 1.2: The minimal-denominator index is reversed

Page 8 · final paragraph before the covering conclusion · arXiv:1502.07648v3

Suspected typo

If n is minimal with $q \le \kappa\rho_n^{-1}$ and $\rho_n = 2^{-n}$, minimality gives

$$q > \kappa\rho_{n-1}^{-1} = \frac{\kappa}{2}\rho_n^{-1},$$

not $q \ge \kappa\rho_{n+1}^{-1}$. The printed version is incompatible with the defining upper bound. The next displayed estimate $q^{-c'} \le (2/\kappa)^{c'} \rho_n^{c'}$ is exactly the consequence of the corrected $n-1$ inequality, so the proof already proceeds with the intended index.

Section 3.4: The Hausdorff–Cantelli cost calculation closes after the repaired neighborhood lemma

Pages 8–9 · proof of Theorem 1.2 · arXiv:1502.07648v3

Correct and complete after the apparently sound repairs above

Maximal separated sets cover the target limsup and have bounded multiplicity under the Ahlfors-regular measure. Corollary 3.8 supplies one affine hyperplane for the relevant rational points on each scale. Applying the repaired Corollary 3.7 with relative thickness ρ_n^{c-1} yields the exponent $c(s-\delta) + \beta(c-1)$, and its positivity is equivalent to $s > \delta - \beta(1 - 1/c)$. No endpoint or quantifier is lost when $c = 1$, and taking arbitrary $\beta < \alpha$ gives the claimed limit. The exact arXiv version was subsequently published in Selecta Mathematica 24 (2018), 3875–3888, DOI 10.1007/s00029-018-0446-7.

Verifiable source: Published article record

03 / NOVELTY

Novelty

No novelty concerns found

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Suspected typographical errors and minor formal issues remain fully visible in yellow with their locations and proposed repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs result from green (no issues found).

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Exact arXiv version 3
2. Published article record

arXiv acknowledgement

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APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.

- Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.

- Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. Contains wrong statements, if at least one central statement is Incorrect.

2. Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.

3. Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.

- Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.

- Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.

- Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.

- Plausible repair only: describe the possible repair but do not treat it as established.

- No repair supplied.

Assign the overall proofs status by the following priority:

1. Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.

2. Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.

3. Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(\delta)$ is proved for every sufficiently small $\delta > 0$ and P is immediately monotone in the required direction, then the claim that $P(\delta)$ holds for every $\delta > 0$ requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use `$...$` for inline mathematics and `$$...$$` for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.