MATHAUDIT / DETAILED REPORT

Correctness, proof completeness, and novelty audit

Khintchine's singular systems and their applications

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Paper	arXiv:0912.4503v1
Source version	Submitted 22 December, 2009; announced December 2009.
Audit date	August 20, 2026
Prompt	Standardized prompt
Novelty cutoff	December 22, 2009
Paper license	arXiv non-exclusive distribution license
Scope	Statements, proofs, and novelty only

Important limitation. AI-generated analysis. This report is not a correctness certificate or a substitute for expert scrutiny. A Correct status may coexist with yellow typos or minor formal corrections that do not affect substantive soundness. It means that this audit found no unresolved substantive error under the stated criteria; it is not formal verification and should remain open to expert challenge and correction.

SUMMARY

01	Statements	Contains unsupported statements
02	Proofs	Contains incorrect or incomplete proofs
03	Novelty	No non-novelty findings

01 / STATEMENTS

Statements

Contains unsupported statements

This paper explicitly presents itself as a survey and attributes most reported results to named sources. Among the claims proved or derived in the paper, Theorem 12 and the exponent refinements in Section 5 are supported after mechanical notation corrections. Theorem 68 is not verified: only a sketch for dimension three is supplied, while the statement covers every dimension at least three.

Surveyed results: Cited theorems are distinguished from present-paper claims

Pages 1–62 and 64–78 · attributed historical and recent results · arXiv:0912.4503v1

Correctly treated as cited results

The abstract and introduction identify the article as a survey. Classical and modern results are normally accompanied by an author's name, a bibliography reference, or explicit wording that the result was announced elsewhere. Their omission of reproduced proofs is therefore not a proof defect of this paper. The audit's proof findings below concern the results for which this paper itself supplies a proof or derivation.

Theorem 12: Almost-every extension preserves the eventual best-approximation sequence

Pages 17–19 · Theorem 12, condition (37), and proof · arXiv:0912.4503v1

Correct

For each competing integer vector with a nonzero added-coordinate block, the exceptional parameters form a product of n slabs. Summing their measures over the original coordinates, added coordinates, and bounded integer shifts gives

$$\zeta_\nu^n M_{\nu+1}^{\max(m+n,m^*)}(\log M_{\nu+1})^{\delta(m^*,m+n)},$$

exactly the summand in condition (37). Borel–Cantelli then excludes all but finitely many competitors for almost every extension. Competitors whose added block vanishes are handled by the original best-approximation property. The notation corrections recorded below restore the intended exceptional set without changing the estimate.

Section 5 exponent refinements: The determinant and rational-plane arguments support the new bounds

Pages 28–36 · Theorems 21, 23, and 25 and their proofs · arXiv:0912.4503v1

Correct

For the cases $(m,n) = (1,3),(2,2),(3,1)$, the proofs select maximal blocks of best approximations in a rational two-plane. The nonzero four-vector determinant gives the global alternative, while the covolume comparison inside the plane gives the complementary alternative. Substituting a power majorant and optimizing the auxiliary growth exponent yields the displayed functions g_1, g_3, and g_2. The determinant-row and function-name slips listed below are uniquely recoverable from their surrounding formulas.

Theorem 68: The arbitrary-gap singular-system construction is not fully proved

Pages 62–64 · Theorem 68, Lemma 8, and proof sketch · arXiv:0912.4503v1

Not able to verify

The theorem asserts the construction for every $m \geq 3$, every prescribed rapidly decreasing ψ, and every prescribed rapidly increasing integer sequence $\tau(\nu)$. The text explicitly explains only the case $m = 3$. Even there, Lemma 8 is supported only by a scheme that selects points inside a rational two-plane and then states that sufficiently small sector parameters make them exactly all ambient best approximations. No quantitative exclusion of integer competitors from the other lattice layers is supplied. The general case $m > 3$ is not addressed. These are central, nontrivial steps, and no complete repair was verified.

02 / PROOFS

Proofs

Contains incorrect or incomplete proofs

The paper's Borel–Cantelli and exponent-inequality proofs close after uniquely determined notation repairs. The construction behind Theorem 68 remains incomplete both in its dimension-three extension lemma and in the passage to arbitrary dimension.

Proof of Theorem 12: The exceptional-set sum matches the convergence hypothesis

Pages 18–19 · Equations (39)–(41) · arXiv:0912.4503v1

Correct and complete

After separating the zero added-coordinate block, a fixed competitor contributes $O(\zeta_\nu^n/\|x_{m+1:m^*}\|^n)$ in parameter measure. There are $O(M_{\nu+1}^{m+n})$ choices of the original coordinates and integer shifts. Summing the added block by its maximum norm produces a power unless $m^* = m+n$, when it produces the single logarithm encoded by $\delta(m^*, m+n)$. Thus the tail measure tends to zero under (37), which is precisely what the Borel–Cantelli step requires.

Theorem 12 notation: Several symbols in the exceptional-set setup are mechanically reversed or omitted

Pages 17–19 · Theorem 12 and Equations (39)–(40) · arXiv:0912.4503v1

Typo

Equation (39) must read $\sum_i \theta_j^i x_i + y_j$, and its minimization must exclude both adjacent lifted best approximations, not only the first. The point vector must contain generic y_j, not $y_{1,\nu}$; the theorem's final coordinate is $y_{n,\nu}$, not $y_{n.\nu}$. Both occurrences of J_ν require minus signs on every original term. Finally, $\Omega_\nu(x, y)$ must use membership in J_ν for every row: it is the bad set where all errors are small, and the following small-measure bound is exactly for those slabs. The printed non-membership sign describes the complementary good set and cannot have the estimated measure. The stated goal and the immediately following estimate uniquely determine all these corrections.

Section 5 proofs: The determinant/covolume case splits cover the required alternatives

Pages 28–36 · proofs of Theorems 21, 23, and 25 · arXiv:0912.4503v1

Correct and complete

Each proof divides the growth of consecutive best-approximation norms into complementary ranges. A nonzero integral determinant supplies the first bound; constancy of the rank-two lattice covolume across the intervening block supplies the other. The monotone majorants requested in the text can be chosen without changing the limiting exponent. The optimization equations defining g_1, g_3, and g_2 then give the displayed lower bounds.

Section 5 notation: Three local formulas contain uniquely identifiable symbol slips

Pages 28 and 32 and 34 · setup and proofs of Theorems 21, 23, and 25 · arXiv:0912.4503v1

Typo

The polynomial displayed for the endpoint α_0 has a repeated x term inconsistent with the immediately stated root interval and comparison and must be read with its intended constant term. In the determinant for the $(2, 2)$ case, the third row's last entry must be $y_{2,k}$ rather than $x_{2,k}$; the same proof's maximum runs over the two forms, not three. In the $(3, 1)$ setup, $h(\alpha) = \alpha - g_2(\alpha) - 1$, not the expression with undefined $g(\alpha)$. Each correction is fixed by the theorem's dimension or by the function defined on the same page and has no effect on the status.

Lemma 8: Ambient best-approximation exclusion is only asserted

Pages 63–64 · Lemma 8 and its proof scheme · arXiv:0912.4503v1

Incomplete as written · no verified repair supplied

The sketch chooses z_{c+1} in a neighboring lattice layer and $z_{c+2}, \ldots, z_d$ as best approximations relative to a rational two-plane. It then says that sufficiently small η_*, δ_* make these exactly the first d best approximations for every parameter in the new sector. Establishing that assertion requires uniform separation from every competing point in all other lattice layers and preservation of the prescribed ordering throughout the sector. Neither a quantitative separation bound nor a finite-competitor reduction is given. The cited similar construction is not identified as a theorem that directly supplies this lemma.

Proof of Theorem 68: Only the case of three variables is discussed

Pages 62–64 · sentence preceding (107) and the induction after Lemma 8 · arXiv:0912.4503v1

Incomplete as written · no verified repair supplied

The proof explicitly begins by restricting its explanation to $m = 3$, and its sectors, rational hyperplanes, and enumeration argument all live in $\mathbb{R}^3$ or $\mathbb{R}^4$. No reduction from general $m \geq 3$ to this case and no higher-dimensional analogue of Lemma 8 is supplied. Since the theorem quantifies over every such m, the omitted passage is a substantive proof obligation rather than a technical detail.

Theorem 69: The following independence theorem is explicitly an external announcement

Page 64 · introduction and statement of Theorem 69 · arXiv:0912.4503v1

Correctly treated as cited, not as proved here

The paragraph immediately preceding Theorem 69 says that it is an assertion announced by Moshchevitin and German in reference [113]. The survey does not present it as a theorem proved in this paper, so the absence of a proof here is not counted as a defect in the paper's own proofs.

03 / NOVELTY

Novelty

No non-novelty findings

No non-novelty findings.

METHOD

Scope and decision rules

This audit addressed only the correctness of central statements, the correctness and completeness of proofs, and evidence-backed non-novelty. It did not evaluate importance, exposition, style, suitability for publication, or author reputation.

An adverse finding is included only where the report can give a direct calculation, counterexample, formal contradiction, or a verifiable source establishing the precise claim. An inability to verify is kept distinct from a demonstrated defect. Technical compression is not treated as incompleteness when the omitted reasoning is recoverable and correct.

Before any finding was recorded, the argument was closed under immediate, mechanically checkable consequences of facts already established. An observation was suppressed when the printed conclusion followed by elementary logic, direct substitution, weakening, inclusion, or verified monotonicity and no substantive proof obligation or textual correction remained.

Typographical errors and minor formal corrections remain fully visible in yellow with their locations and repairs. When such a defect is unambiguous, local, and harmless to the paper's substantive conclusions, it does not lower the overall statements or proofs status from Correct.

The novelty panel reports only central results for which a verifiable primary source predating the earliest public version that contains the result proves the same statement up to notation, or from which the statement follows directly and immediately. It contains no positive novelty certifications and no entries based merely on an unsuccessful literature search.

All mathematical expressions in this report are compiled by LaTeX. The web presentation independently validates and renders the corresponding expressions with KaTeX.

Sources checked

1. Source paper: arXiv:0912.4503v1

arXiv acknowledgement

Thank you to arXiv for use of its open access interoperability. This service was not reviewed or approved by, nor does it necessarily express or reflect the policies or opinions of, arXiv.

APPENDIX

Standardized prompt

You are a mathematical auditor. Evaluate only the correctness of the paper's central statements, the correctness and completeness of its proofs, and evidence-backed non-novelty. Do not make editorial comments or evaluate importance, exposition, style, or suitability for publication. Keep the report neutral, precise, and professional.

PART 1 — CORRECTNESS OF THE MAIN STATEMENTS

Identify the paper's central statements and assess each one separately using exactly one of these outcomes:

- *Correct: the statement has a correct, verifiable proof; or, despite a defect in the printed proof, you can give a fully verified repair or independent proof.*
- *Incorrect: you can give strong formal evidence that the statement as written is false, preferably a counterexample or a complete contradiction.*
- *Not able to verify: the supplied proof does not establish the statement, and you can neither supply a fully verified proof nor disprove it.*

For every adverse or unverified outcome, restate the exact claim and hypotheses, identify its location in the reviewed version, and give the decisive reasoning. Do not infer that a theorem is false merely because its printed proof is defective.

Assign the overall statements status by the following priority:

1. *Contains wrong statements, if at least one central statement is Incorrect.*
2. *Contains unsupported statements, if no central statement is Incorrect but at least one is Not able to verify.*
3. *Correct, if every central statement is Correct, apart from any typographical or minor formal findings allowed by Part 4.*

PART 2 — CORRECTNESS AND COMPLETENESS OF THE PROOFS

Audit the proofs of the central statements and every material internal or cited input on which they depend. For each proof or proof segment, use the narrowest supported outcome:

- *Correct and complete: every substantive step is verifiable and no case or crucial nontrivial step is missing.*
- *Incorrect as written: a displayed calculation, inference, lemma, or cited application is formally false or inapplicable.*
- *Incomplete as written: a necessary substantive step or case is missing, even though no explicit contradiction has been proved.*
- *Not able to verify: the available paper and sources are insufficient to decide whether the step is valid.*

Do not flag omitted routine details, compressed arguments, or "analogously" steps when the omitted reasoning is recoverable and correct. For every defect, identify its smallest exact location, explain its downstream dependency, and classify any proposed repair as one of:

- *Verified repair: supply enough reasoning to verify that the repair works throughout every affected use.*
- *Plausible repair only: describe the possible repair but do not treat it as established.*
- *No repair supplied.*

Assign the overall proofs status by the following priority:

1. *Contains incorrect or incomplete proofs, if at least one proof is Incorrect as written or Incomplete as written.*
2. *Contains unverified proofs, if there is no proved incorrectness or incompleteness but at least one material proof is Not able to verify.*
3. *Correct, if all audited proofs are Correct and complete, apart from any typographical or minor formal findings allowed by Part 4.*

PART 3 — NOVELTY OF THE MAIN STATEMENTS

This is a one-sided, evidence-only check. Evaluate only central results. Report a result only when a verifiable primary source, publicly available before the earliest public version of the paper that contains the result, proves the same statement up to a change of notation, or when the result follows directly and immediately from such an earlier theorem.

For each non-novelty finding, provide the earlier source, its public date, the exact theorem or proposition, and an explicit hypothesis-and-conclusion mapping. A later source cannot establish non-novelty. Do not flag a result merely because an easier proof exists or because it follows through a nontrivial argument. Do not flag a result that the paper itself explicitly presents as known, a survey item, or an alternative proof.

Mention only statements that meet this non-novelty threshold. Say nothing about statements for which no such evidence was found: do not certify novelty, list unsuccessful searches, or add statement-level "no prior source found" entries. If the list is empty, the section may say only "No non-novelty findings."

PART 4 — TECHNICAL AND EVIDENTIARY RULES

1. Criticism must never be overstated. Call a statement incorrect, a proof incorrect or incomplete, or a result non-new only when strong formal evidence supports that exact judgment. Otherwise use Not able to verify where that outcome exists, or remain silent in the novelty section.

2. Review a specified paper version. Give exact page, theorem, equation, or proof locations; restate relevant quantifiers and hypotheses; and check domains, endpoints, parameter ranges, constants, and dependencies explicitly. Apply a defect only to the regimes and downstream results that actually depend on it.

3. Immediate-consequence closure: when deciding whether a conclusion is supported, treat the argument as including every immediate formal consequence of facts established earlier in the paper and available at that point. An immediate formal consequence is a local, mechanically checkable inference using only definitions already in force, elementary logic, direct substitution or renaming, weakening of an inequality or set inclusion, or a monotonicity or symmetry property explicit in a definition or already proved.

4. Such an inference must introduce no new lemma, estimate, construction, case analysis, limiting or compactness argument, external result, or new issue of uniformity or parameter dependence. Check the direction of monotonicity and the dependence of every threshold or constant explicitly. Do not exchange pointwise and uniform conclusions, change exceptional sets, interchange quantifiers or limits, or extend an endpoint under this rule unless the required fact was already established.

5. If the claimed conclusion follows after immediate-consequence closure, it is fully supported. Do not report a gap, unsupported statement, minor formal correction, or recommendation to add an explanatory sentence, and do not mention the suppressed item in the report. This suppression does not apply to a genuine typographical error as defined below, because a typo is a literal notation defect that still requires a textual correction. For example, if $P(delta)$ is proved for every sufficiently small delta > 0 and P is immediately monotone in the required direction, then the claim that $P(delta)$ holds for every delta > 0 requires no correction; the omitted monotonicity step is exposition rather than an audit finding.

6. Immediate-consequence closure permits reconstruction of an omitted inference; it does not permit silent alteration of the printed domain, hypotheses, or quantifiers of an intermediate statement later invoked outside its stated scope. If formal validity requires changing the text of that statement or its stated range, report the appropriate finding even when the repair is easy. Thus a result stated only for $t > 0$ and subsequently applied at $t = 0$ remains a formal scope mismatch unless the endpoint case was separately established before the application.

7. Apply this mandatory gate before recording any adverse, unverified, typographical, or minor formal finding: (a) identify the exact assertion or inference alleged to be defective; (b) reconstruct the strongest conclusion following from the preceding argument after all admissible immediate formal consequences are added; (c) identify the specific defect that remains — a formally demonstrated falsehood, a concrete nontrivial unresolved proof obligation, or a genuine domain, hypothesis, quantifier, or notation mismatch requiring a textual correction; and (d) state the evidence and minimal repair. If no specific defect remains, or if the only possible addition would be explanatory, suppress the finding entirely. Use Not able to verify only when a precise, nontrivial unresolved obligation can be identified.

8. A typographical error is an incorrect symbol, index, exponent, sign, variable name, or notation mismatch whose unique intended correction is mechanically determined from the surrounding text and does not alter the mathematical argument. Report it in yellow as Typo, with the printed text, correction, exact location, and reason it is harmless. It does not lower an overall status.

9. A minor formal correction is a boundary, endpoint, or displayed parameter-range defect that can be repaired consistently in every affected statement by a local change, without a new argument, without invalidating any related result or application, and without creating another substantive defect. Report it in yellow as Minor formal correction, with the same details. It does not lower an overall status.

10. Do not use either minor category if more than one materially different repair is possible; if the repair changes the substance or scope of a central result; if an excluded case is used later; if the correction fails to propagate consistently; or if a new mathematical idea is required. In such cases use the strongest substantive classification actually supported by evidence.

11. Write every mathematical expression as valid, compilable LaTeX. Use ... for inline mathematics and

...

for displayed mathematics, with balanced delimiters and braces. Do not leave raw LaTeX commands outside math delimiters.

12. Separate theorem truth from the correctness of the printed proof. A theorem can be Correct in Part 1 because of a fully verified independent proof while its printed proof is adverse in Part 2. Conversely, a defective proof alone never licenses an Incorrect theorem classification.

Organize the report under Parts 1–3 and apply Part 4 throughout. Do not add editorial commentary, scores, rankings, recommendations, or a list of suppressed observations.

END OF REPORT

Each claim in this prototype report remains open to independent challenge and revision.